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Real Functions Operating on Sub-lattices of $C(X)$

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Let F be a vector- lattice of real functions on a set X and let $\mathrm{cl}(F)$ be its uniform closure. This talk is devoted to the study of conditions on F, in order that $\mathrm{cl}(F)$ has certain algebraic properties.

Specifically, we analyse the problems of knowing when $\mathrm{cl}(F)$ is closed under composition with all the real uniformly continuous functions over $\mathbb{R}$, when it is a ring, or it is closed under composition with all the continuous function over $\mathbb{R}$, or with all the continuous function over the open sets of $\mathbb{R}$.

It will be noticed that, if F contains unbounded functions, each one of these problems is different to each other. For instance, if $\mathrm{cl}(F)$ is a uniformly closed ring, then it is also closed under composition with the functions of the ring generated by the polynomials and the functions of $C(\mathbb{R})$ that vanish at infinity, but not under all the functions in $C(\mathbb{R})$. Nevertheless, most of the results presented here have been obtained by applying a common technique that involves certain kind of countable covers of X, the so-called 2-finite covers.

References

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