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Selectors and topological properties of spaces

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We study continuous selectors $\sigma : \mathfrak{F}_\tau(X) \to X$ where $\mathfrak{F}_\tau(X)$ is a hyperspace of non-empty closed subsets of X equipped with a topology τ and $\sigma(F) \in F$ for each $F \in \mathfrak{F}(X)$. This topic has been investigated for years and we are going to mention some new results and to relate them to other results in the literature.

Some sample facts from our background:

THEOREM 1 (Mazurkiewicz-Sierpiński). *X is first countable scattered compact $\iff$ X is homeomorphic to a countable ordinal $\iff$ X is compact and countable.*

A zero-selector on X is a selector σ such that $\sigma(F)$ is isolated relatively to F.

If X has a zero-selector, then X is scattered. Moreover, any subspace of ordinals has a zero-selector–just take minima of non-empty closed sets.

A first-countable paracompact scattered space is a completely metrizable subspace of an ordinal space (Telgársky); consequently it has a zero-selector.

THEOREM 2 (Fujii-Nogura). *If X is compact and there exist a zero-selector, then X is homeomorphic to an ordinal space.*

New and also older results mentioned in this lecture were obtained jointly with G.Artico, U.Marconi, L.Rotter and M.Tkachenko.

References

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