On the Bestvina-Edwards problem

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Poincaré constructed the first example of a homological 3-sphere with a nontrivial fundamental group. The complement of an open 3-ball in this space is an acyclic finite noncontractible polyhedron P. It follows by the Mayer-Vietoris sequence and the van Kampen theorem that the suspension ΣP of this polyhedron is an acyclic space with the trivial fundamental group. It follows by the Hurewicz theorem that the suspension ΣP has all homotopy groups trivial and is hence a contractible space. Complex P is an acyclic noncontractible compactum. Every cell-like space is acyclic in Čech cohomology and every contractible compactum is clearly cell-like. So there is a natural question: Does there exist a noncontractible cell-like compactum whose suspension is contractible? (M.Bestvina-R.D.Edwards, Problem D28 in J.van Mill and G.M.Reed, *Open Problems in Topology*, North-Holland, Amsterdam 1990). Earlier we have proved (with U. H. Karimov) that there exists a noncontractible cohomologically locally connected (clc) 2-dimensional compact metric space X of trivial (Borsuk) shape whose reduced suspension is a contractible absolute retract. However, the unreduced suspension of X turned out to be noncontractible, so the question remained open. In this talk I shall present our new result - we have proved that the answer to the Bestvina-Edwards question is affirmative. I shall also state some interesting applications and related open problems.