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Trigonometrical Identities and Isoperimetric Inequality

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The main result of this report is the following:

THEOREM 1. *Let* $2 < m_0, n_0 < 4$ *real roots of the equation* $f(m_0, n_0) = 0$*, where*

$$\begin{aligned} f(m,n) \quad = \quad & -2 + 6(M_m + M_n) - 6(M_m^2 + M_n^2) + 2(M_m^3 + M_n^3) + 65M_mM_n + \\ & 32(M_m^2M_n + M_mM_n^2) + 26M_m^2M_n^2 + 5(M_m^3M_n + M_mM_n^3) + \\ & 4(M_m^3M_n^2 + M_m^2M_n^3) + M_m^3M_n^3 \end{aligned}$$

and $M_k = \cot^2(\pi/k)$.
Then there exists a 32-faced polyhedron in the 3-dimentional Euclidean space which is the canonical fundamental set of the Euclidean cone-manifold $W(m_0, n_0)$.

A.D. Mednykh (1999) established the Tangent and the Sine Rules relating the complex lengthes of the singular geodesics and the cone angles of $W(m,n)$ (Theorems A and B).

In this report the Euclidean analogues of the Theorems A and B are obtained.

THEOREM 2 (The Euclidean Tangent Rule). *Let* $\gamma_m = l_m + i\,\varphi_m$ *be a complex length of the singular geodesic of the hyperbolic cone-manifold* $W(m,n)$ *with cone angle* $2\pi/m$*. If* $W(m,n)$ *admits the Euclidean structure, then*

$$\frac{\tan\frac{\varphi_m}{4}}{\tan\frac{\varphi_n}{4}} = \frac{\tan\frac{\pi}{m}}{\tan\frac{\pi}{n}}.$$

THEOREM 3 (The Euclidean Sine Rule I). *Let* $\gamma_m = l_m + i\,\varphi_m$ *be a complex length of the singular geodesic of the hyperbolic cone-manifold* $W(m,n)$ *with cone angle* $2\pi/m$*. If* $W(m,n)$ *admits the Euclidean structure, then*

$$\frac{\sin\frac{\varphi_m}{2}}{l_m} = \frac{\sin\frac{\varphi_n}{2}}{l_n}.$$

Moreover, in this report the following theorems are established.

THEOREM 4 (The Euclidean Sine Rule II). *Under condidions of Theorem 3 we have*

$$\frac{\sin\left(\frac{2\pi}{m} - \frac{\varphi_m}{2}\right)}{\sin\frac{\varphi_n}{2}} = \frac{\sin\left(\frac{2\pi}{n} - \frac{\varphi_n}{2}\right)}{\sin\frac{\varphi_m}{2}}.$$

THEOREM 5. *If* $W(m,n)$ *admits the Euclidean structure, then*

$$Vol\ W(m,n) = \frac{1}{12}\left(\frac{4-\sin^2\theta}{\sin\theta}\right) l_m l_n a,$$

where $Vol\ W(m,n)$ is the volume, a is the shortest distance and θ is the acute angle between the components of the singular set of $W(m,n)$.

THEOREM 6 (Isoperimetric Inequality). *Under conditions of Theorem 5 we have*

$$0 < Vol\ W(m,n) < 0,29\ l_m l_n a.$$

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