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Compact and countably compact versus sequentially compact

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It is well-known that a compact Hausdorff space can be very far from being sequentially compact. Basic examples of this kind are the Tychonoff cube $I^{\mathfrak{c}}$ and the Cech-Stone compactification $\beta\omega$ of the discrete space ω. These two spaces have the common feature to be "big" as both of them have cardinality $2^{\mathfrak{c}}$.
One may wonder whether some cardinal restriction is sufficient to guarantee the sequential compactness and in fact many authors have already published results of this sort. For instance, in 1969 Franklin showed that a compact Hausdorff space of cardinality less than 2^{ω_1} is always sequentially compact and in 1976 Levyn showed that a compact T_1-space of cardinality not exceeding ω_1 is always sequentially compact. Later on, these two results have been improved by replacing ω_1 with the cardinal $\mathfrak{t}$: in 1984 van Douwen for the Hausdorff case and in 2005 Alas-Wilson for the T_1 case.
Our aim is to continue this line of investigation. A prominent role here is played by the following cardinals:
$\mathfrak{t}$, that is the smallest cardinality of a tower in $[\omega]^{\omega}$;
$\mathfrak{h}$, that is the smallest height of a tree π-base in ω^*;
$\mathfrak{n}$, that is the smallest cardinality of a cover of ω^* by nowhere dense sets.

(This is a joint work with P. J. Nyikos.)