



ITES2007 Sixth Italian-Spanish conference
on General Topology and applications

Bressanone, 26-29 June 2007

When is a uniformity determined by its Hausdorff hypertopology?

Ahmed Bouziad

Laboratoire Université de Rouen

Technopole du Madrillet, F76801 Saint-Etienne du Rouveray, France

ahmed.bouziad@univ-rouen.fr

Two uniformities \mathcal{U} and \mathcal{V} on a set X are said to be H -equivalent if their corresponding Hausdorff uniformities on the set of all non-empty subsets of X induce the same topology. The uniformity \mathcal{U} is said to be H -singular if no distinct uniformity on X is H -equivalent to \mathcal{U} . The self-explanatory concepts of H -coarser, H -minimal and H -maximal uniformities are defined similarly.

In 1966, J. Isbell, A. Ivanov and A.J. Ward exhibited (independently) examples showing that not all uniformities are H -singular. Here, we show that there is a property which obstructs H -singularity: Every H -minimal uniformity has a base of finite-dimensional covers.

It is also well-known that proximally equivalent uniformities are just those with the same precompact reflections. We study the analogous situation for H -equivalence and show that the corresponding reflection exists. In fact, the germ of this reflection can be seen in the work of A.J. Ward; here, the construction is achieved in detail and the relevant universal properties are established. Furthermore, we give an intrinsic characterization of H -minimal uniformities and show that they are H -coarser. This characterization of H -minimality becomes a criterion for H -singularity for proximally fine uniformities (hence, in particular, for metrizable ones) and for all uniformities that are either complete or uniformly locally precompact.

Some relevant properties which insure H -singularity are introduced and investigated in detail.