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Properties transfer between hyperspaces and function spaces

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Many known convergences (e.g., continuous convergence, Isbell topology, compact-open topology, pointwise convergence) on the space of continuous maps (valued in a topological space) can be represented as the dual convergences with respect to collections of families of sets. The sets $[\mathcal{A}, O] := \{f \in C(X,Z) : f^{-}(O) \in \mathcal{A}\}$, where $\mathcal{A} \in \alpha$ and O is an open subset of Z, is a subbase of the *dual topology* $\alpha(X,Z)$ of a collection α of families of open subsets of X. In particular, for the *Sierpiński topology* \$, the topology $\alpha(X,\$)$ is defined on the hyperspace $C(X,\$)$ of X. The following characterization is fundamental for clarifying the relationship between convergences on spaces of real-functions and of the corresponding hyperspaces:

$$f \in \lim_{\alpha(X,Z)} \mathcal{F} \Leftrightarrow \forall_{C \in C(X,\$)}\ f^{-1}(C) \in \lim_{\alpha(X,\$)} \mathcal{F}^{-1}(C),$$

where $\mathcal{F}^{-1}(C)$ is the filter generated by $\{f^{-1}(C) : f \in F\}$, with $F \in \mathcal{F}$.

The γ-connection initiated by G. Gruenhage and developed by F. Jordan to characterize many properties of $C_p(X,\mathbb{R})$ in terms of X is generalized to arbitrary $C_\alpha(X,Z)$ (the space $C(X,Z)$ endowed with $\alpha(X,Z)$) and in particular to open-set topologies. This characterization involves neighborhood filters in domains of open sets. Consequently, it can be transferred with the aid of complementary convergences to hyperspace topologies.

References

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