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On some cardinal invariants of compact and H-closed spaces

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1. It is well-known that for T_2-compact spaces the following is true: $t(X) \leq \psi(X) = \chi(X)$. Here $t(X)$, $\psi(X)$, $\chi(X)$ denote tightness, pseudocharacter, character of a space X. For T_1-compact spaces we can only say: $t(X) \leq \chi(X)$, $\psi(X) \leq \chi(X)$. It is not true in general for T_1-compact spaces that $t(X) \leq \psi(X)$. But we prove

Theorem 1. *$t(X) \leq \psi(X)$ for T_1-selfconjugative space X.*

A space X is called selfconjugative if $F \subset X$ is compact iff F is a closed set (A. Arhangelskii, [**1**]).

2. From now all spaces are T_2-spaces. In [**4**] the author proved the theorem:

(*) $|X| \leq 2^{\chi(X)}$ *for H-closed X and $\chi(X) \leq \omega$.*

Because of some difficulties we could not prove the theorem for every $\chi(X)$. Some later and by other method it was proved by A. Dow and J. Porter [**3**]. Now we prove

Theorem 2. *Let $\tau = \chi(X)$, X is a minimal space. Then for every $A \subset X$, $|A| \leq 2^\tau$, there is $F \subset X$, $|F| \leq 2^\tau$, $A \subset F$ and F is H-set and τH-closed.*

A set F is called a H-set if for every open cover $\gamma = \{U\}$ of F there is a finite $\gamma' \subset \gamma$ such, that $F \subset \{[U] : U \in \gamma'\}$.

A space X is called τH-closed if, for every open cover $\gamma = U$, $|\gamma| \leq \tau$ of X there is a finite $\gamma' \subset \gamma$ such, that $X = \{[U] : U \in \gamma'\}$.

This theorem enables us to overcome difficulties, mentioned above, and to prove the inequality $|X| \leq 2^{\chi(X)}$ for every $\chi(X)$ by obvious modification of the proof of (*).

References

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