



On Paracompact and Locally Compact Paracompact Extensions

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By a “topological space” we will understand a “Hausdorff topological space”. As the celebrated Smirnov Compactification Theorem [7] shows, the compactifications of a Tychonoff space can be described by means of Efremovič proximities. Our purpose in this talk is to answer the following two natural questions: are there proximity-type descriptions of the ordered sets of all (up to equivalence) paracompact (resp., locally compact paracompact) extensions of a Tychonoff space? We will give affirmative answers to the above questions. We will do this using the notion of *SR-proximity* which was introduced in [5] and was exploited there for obtaining a description of the ordered set of all (up to equivalence) regular extensions of a regular space and for descriptions of some other kinds of extensions. There are many descriptions of the paracompact (resp., locally compact paracompact) extensions of a Tychonoff space, but all of them are not of a proximity type. As far as we know, the first description of paracompact extensions was given by H. Bentley and H. Herrlich in [2]; they used some special nearness structures (see also the paper of H. Bentley [1]). In [10], V. Zaicev gave another description by means of projection spectra. Finally, in [3], A. Borubaev obtained a description using uniform structures. The locally compact paracompact extensions were described by V. Zaharov in [8, 9] by means of some special vector lattices of functions, by D. Doitchinov in [6] – on the basis of the notion of *supertopological space*, and by A. Borubaev [4] – using uniform structures.

References

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