ITES2007 Sixth Italian-Spanish conference
on General Topology and applications

Bressanone, 26-29 June 2007

$\mathcal{S}$-topologies and bounded convergences

Jesús Rodríguez-López

E.T.S.I. de Caminos, Canales y Puertos
Departamento de Matemática Aplicada – Instituto de Matemática Pura y Aplicada
Universidad Politécnica de Valencia, 46022 Valencia, Spain.

jrlopez@mat.upv.es

The study of convergences of closed subsets of a topological space has played an important role in several mathematical branches: functional analysis, optimization theory, variational analysis, etc. (see [**1**]). Traditionally, given a topological space (X, τ) the topologies defined on a family of subsets of X have been called *hypertopologies.*

The origin of hyperspace theory is due to Hausdorff when he defined the so-called Hausdorff distance. Given a metric space (X, d) the family $\mathcal{CLB}_0(X)$ of all nonempty closed bounded subsets of X can be endowed with the following metric

$$H_d(A, B) = \max\{\sup_{a \in A} d(a, B), \sup_{b \in B} d(A, b)\}.$$

We are interested in the fact that convergence in the topology generated by the Hausdorff metric H_d can be expressed in the following terms:

- $\{A_\lambda\}_{\lambda \in \Lambda}$ is convergent to A if and only if $\{d(A_\lambda, \cdot)\}_{\lambda \in \Lambda}$ is uniform convergent to $d(A, \cdot)$;
- $\{A_\lambda\}_{\lambda \in \Lambda}$ is convergent to A if and only if for all $\varepsilon > 0$, $A_\lambda \subseteq B_d(A, \varepsilon)$ and $A \subseteq B_d(A_\lambda, \varepsilon)$ residually,

where $B_d(A, \varepsilon) = \{x \in X : \text{ there exists } a \in A \text{ such that } d(a, x) < \varepsilon\}$.

Other hypertopologies follow one of the above two models. For example [**1**], a net $\{A_\lambda\}_{\lambda \in \Lambda}$ is convergent to A in the Attouch-Wets topology if and only if for all $\varepsilon > 0$ and for all bounded subset S we have that $A_\lambda \cap S \subseteq B_d(A, \varepsilon)$ and $A \cap S \subseteq B_d(A_\lambda, \varepsilon)$ residually. Furthermore, it can be proved that this is also equivalent to uniform convergence of distance functions on bounded sets.

Recently, Lechicki, Levi and Spakowski [**2**] have studied set convergence of the Attouch-Wets type expressed in terms of truncations by members of a prescribed family of sets $\mathcal{S}$. A net $\{A_\lambda\}_{\lambda \in \Lambda}$ is $\mathcal{S}$-convergent to A if for each $S \in \mathcal{S}$ and $\varepsilon > 0$, $A_\lambda \cap S \subseteq B_d(A, \varepsilon)$ and $A \cap S \subseteq B_d(A_\lambda, \varepsilon)$ residually. As we have previously observed, when $\mathcal{S}$ is either the family of metrically bounded sets or the family of all nonempty subsets, it is well-known that $\mathcal{S}$-convergence means uniform convergence of distance functions on members of $\mathcal{S}$. In this talk we will show that this coincidence for these two convergences is not true for an arbitrary family. We will give necessary and sufficient conditions for coincidence to occur when the family is a general ideal of subsets.

References

[1] G. Beer, *Topologies on Closed and Closed Convex Sets*, vol. 268, Kluwer Academic Publishers, 1993.

[2] A. Lechicki, S. Levi and A. Spakowski, *Bornological convergences*, J. Math. Anal. Appl. **297** (2004), 751–770.

(This is a joint work with G. Beer and S. Naimpally.)