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On the maximal based-free equivariant compactification

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Let G be a compact Lie group and X be a G-space. Recall that the stabilizer G_x of a point $x \in X$ is defined by $G_x = \{g \in G|\ gx = x\}$. A G-space X is called *free* if $G_x = \{1\}$, the trivial subgroup of G, for all $x \in X$. A G-space X is called semifree if for any $x \in X$ either $G_x = \{1\}$ or $G_x = G$.

If there exists a unique point $a \in X$ such that $G_a = G$ and $G_x = \{1\}$ for every $x \in X \setminus \{a\}$, then X is called a *based-free* G-space. It is a well-known result of R. Palais that every G-space (completely regular and Hausdorff) possesses a Hausdorff G-compactification (or equivariant compactification). On the other hand, not every free G-space X admits a free G-compactification, while such a G-space always admits a based-free G-compactification [**1**]. In this case there exists a unique maximal based-free G-compactification of X denoted by $\beta^*_G X$.

In this paper we show that, in general, $\beta^*_G X$ is not equivalent to the maximal equivariant compactification $\beta_G X$. We also present the following characterization of $\beta^*_G X$:

Let G be a compact Lie group and X be a free G-space. Then:

(1) *Each G-map $f : X \to \mathrm{Cone}(G)$ to the cone over G admits a unique G-extension $F : \beta^*_G X \to \mathrm{Cone}(G)$.*

(2) *If cX is a based-free G-compactification of X such that each G-map $f : X \to \mathrm{Cone}(G)$ extends to a G-map $F : cX \to \mathrm{Cone}(G)$, then cX is equivalent to $\beta^*_G X$.*

References

[1] N. Antonyan, *Equivariant embeddings and compactifications of free g-spaces*, Internat. J. Math. Math. Sci. **1** (2003), 1–14.