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Transversal perturbations of convexity

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The classical Caratheodory's theorem states that for a subset X of a n-dimensional vector space and for a point $p \in \operatorname{conv} X$ the inclusion $p \in \operatorname{conv}\{x_1, x_2, \dots, x_{n+1}\}$ holds for some elements $x_i \in X, i = 1, 2, \dots, n+1$.

We prove a generalization of the result for subsets X of $(n+1)$-dimensional vector spaces which are graphs of continuous functions with convex n-dimensional domains.

Theorem. *Let X be a graph of continuous real-valued function with convex n-dimensional domains. Let $y_1, y_2, \dots, y_{n+2}$ be any points in X and z be a point in the convex hull* $\operatorname{conv}\{y_1, y_2, \dots, y_{n+2}\}$.

Then there exists points $x_1, x_2, \dots, x_{n+1}$ in X such that $z \in \operatorname{conv}\{x_1, x_2, \dots, x_{n+1}\}$ and the simplex $\operatorname{conv}\{x_1, x_2, \dots, x_{n+1}\}$ *can be moved (with respect to some parallel transfer) into one of the faces of the simplex* $\operatorname{conv}\{y_1, y_2, \dots, y_{n+2}\}$.

Some applications to the theory of continuous selections of nonconvex-valued mappings are presented.

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