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Pseudocompact spaces under CH

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A topological space is called *pseudocompact*, if it is Tychonoff and every real-valued continuous function defined on it is bounded.
It is well-known that *compact* $\longrightarrow$ *countably compact* $\longrightarrow$ *pseudocompact* and none of the implications can be reversed.
We wish to discuss the following theorems:

Theorem A. (A. V. Arhangel'skii, A. Bella 1996) *Every countably compact regular space of countable tightness has a countable fan-tightness.*

Theorem B. (A. V. Arhangel'skii, A. Bella 1996) *Every countably compact regular space of countable tightness is discretely generated.*

Theorem C. (A. V. Arhangel'skii 1972) *Every countably compact regular space is Fréchet if and only if it is strongly Fréchet.*

Theorem D. (V. Tkachuk, A. Yaschenko 2001) *Every countably compact regular Whyburn space is Fréchet.*

We shall show that the assumption of countable compactness in all these theorems cannot be weakened to pseudocompactness. We want to describe a universal method, originated from O. Pavlov's ideas, for constructing pseudocompact examples disproving the corresponding weaker statements. The technique requires the Continuum Hypothesis; the existence of such spaces in ZFC is an open problem.

(This is a joint work with Angelo Bella.)