

## 7.a WCET analysis techniques

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### Computing the WCET /1

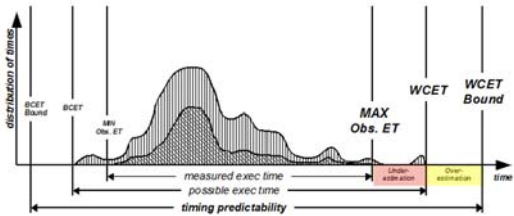
- Why not measure the WCET of a task on its real hardware?
  - Worst-case input → Task
  - Worst-case HW state → Target Hardware (black box) → Logic analyser, oscilloscope, etc. **WCET?**
- Triggering the WCET by test is very difficult
  - ❑ Worst-case input covering all executions of a real program is intractable in practice
  - ❑ Worst-case initial state is difficult to determine with modern HW
    - Complex pipelines (out-of-order execution)
    - Caches
    - Branch predictors and speculative execution

### Worst-case execution time (WCET)

- For any input data and all initial logical states
  - ❑ So that all execution paths are covered
- For any hardware state
  - ❑ So that worst-case conditions are in effect
- **Measurement-based** WCET analysis
  - ❑ On the real HW or a cycle-accurate simulator
  - ❑ The *high-watermark* value can be  $\leq$  WCET
- **Static** WCET analysis
  - ❑ On an abstract model of the HW and of the program

### Computing the WCET /2

- Exact WCET not generally computable ( $\sim$  the *halting problem*)
- A WCET estimate or *bound* are key to predictability
  - ❑ Must be *safe* to be an upper bound to all possible executions
  - ❑ Must be *tight* to avoid costly over-dimensioning



Static WCET analysis /1

- To analyze a program without executing it
  - ❑ Needs an *abstract model* of the target HW
  - ❑ As well as of the actual executable
- Execution time depends on control path and HW
  - ❑ *High-level analysis* addresses the program behavior
    - Control flow analysis builds a control flow graph (CFG)
  - ❑ *Low-level analysis* determines the timing behavior of individual instructions
    - Not constant for modern HW
    - Must be aware of the HW inner workings (pipeline, caches, etc.)

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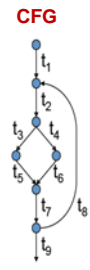
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Implicit path enumeration technique

- The program structure is mapped into flow graph constraints
  - ❑ WCET computed with integer linear programming or constraint-solving techniques
- $WCET = \sum_i x_i \times t_i$ 
  - ❑ Where  $x_i$  is the execution frequency of CFG edge  $i$
  - ❑ And  $t_i$  the execution time of CFG edge  $i$

CFG



Flow constraints

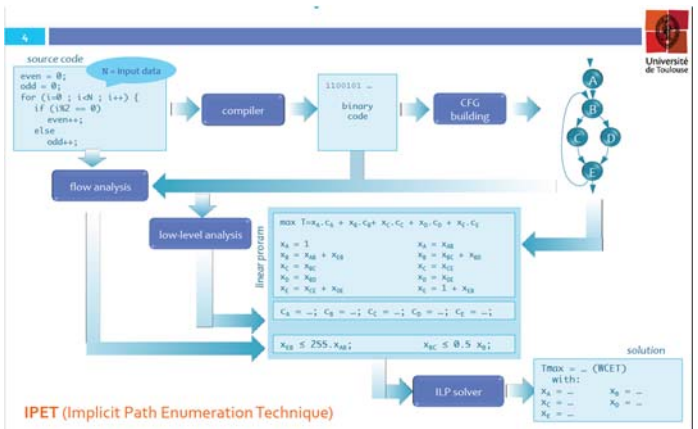
$$\begin{aligned} x_1 &= 1 \\ x_1 + x_8 &= x_2 \\ x_2 &= x_3 + x_4 \\ x_3 &= x_5 \\ x_4 &= x_6 \\ x_5 + x_6 &= x_7 \\ x_7 &= x_8 + x_9 \\ x_2 &\leq LB \times x_1 \end{aligned}$$

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Static WCET analysis /2



source code

```
even = 0;
odd = 0;
for (i=0; i<N; i++) {
  if (i%2 == 0)
    even++;
  else
    odd++;
}
```

compiler

binary code

CFG building

flow analysis

low-level analysis

linear program

$$\begin{aligned} \max \quad & T = x_1 \cdot C_1 + x_2 \cdot C_2 + x_3 \cdot C_3 + x_4 \cdot C_4 + x_5 \cdot C_5 + x_6 \cdot C_6 \\ & x_1 = 1 \\ & x_1 = x_2 + x_3 \\ & x_2 = x_4 + x_5 \\ & x_3 = x_6 \\ & x_4 = x_7 + x_8 \\ & x_5 = 1 + x_6 \\ & C_1 = -1; C_2 = -1; C_3 = -1; C_4 = -1; C_5 = -1; C_6 = -1; \\ & x_1 \leq 255 \cdot x_{\max}; \quad x_{\max} \leq 0.5 \cdot x_1; \end{aligned}$$

H.P. solver

solution

$$\begin{aligned} T_{\max} &= \dots (WCET) \\ \text{with:} \quad & x_1 = \dots \\ & x_2 = \dots \\ & x_3 = \dots \\ & x_4 = \dots \end{aligned}$$

IPET (Implicit Path Enumeration Technique)

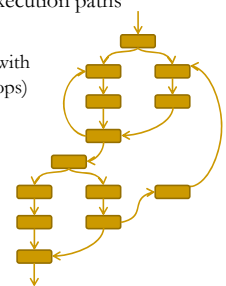
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Static WCET analysis /3

- *High-level analysis* /1
  - ❑ Must analyze all possible execution paths of the program
    - Builds the CFG as a superset of all possible execution paths
    - *Basic block* is the unit of that analysis
      - ❑ The longest sequence of program instructions with single entry and single exit (no branches, no loops)
  - ❑ Challenges with path analysis
    - *Input-data dependency*
    - *Infeasible paths*
    - *Loop bounds* (and recursion depth)
    - *Dynamic calls* (through pointers)




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Static WCET analysis /4

■ **High-level analysis** /2

- Several techniques are deployed to allow using IPET
  - *Control-flow analysis* to construct the CFG
    - First finding the basic blocks and then building the graph among them
  - *Data-flow analysis* to find loop bounds
  - *Value analysis* to resolve memory accesses
- Automatic information extraction is insufficient 
  - User annotation of *flow facts* is needed
    - To facilitate detection of infeasible paths
    - To refine loop bounds
    - To define frequency relations between basic blocks
    - To specify the target of dynamic calls and referenced memory addresses

Static WCET analysis /6

■ **Low-level analysis** /2

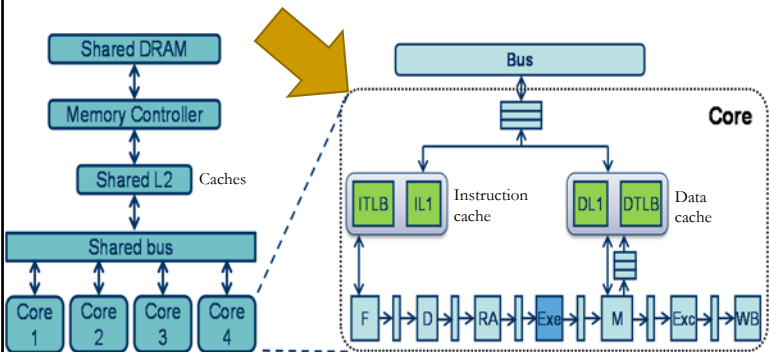
- Concrete HW states
  - Determined by the history of execution
  - Cannot compute all HW states for all possible executions
    - Invariant HW states are grouped into execution contexts
    - *Conservative overestimations* are made to reduce the research space
- *Abstract interpretation*
  - Computes abstract states and specific operators in the abstract domain
    - *Update function* to keep the abstract state current along the exec path
    - *Join function* to merge control flows after a branch
- Some techniques are specific to each HW feature

Static WCET analysis /5

■ **Low-level analysis** /1

- Requires abstract modeling of all HW features
  - Processor, memory subsystem, buses, peripherals, ...
  - It is *conservative* : it must never underestimate actual timing
  - All possible HW states should be accounted for
- Challenges with HW modeling
  - *Precise modeling* of complex hardware is difficult
    - Inherent complexity (e.g., out-of-order pipelines)
    - Lack of comprehensive information (intellectual property, patents, ...)
    - Differences between specification and implementation (!)
  - *Exhaustive representation* of all HW states is computationally infeasible

Understanding the hardware /1



Courtesy of PROXIMA

### Cache Associativity

Just as bookshelves come in different shapes and sizes, caches can also take on a variety of forms and capacities. But no matter how large or small they are, caches fall into one of three categories: direct mapped, n-way set associative, and fully associative.

Direct Mapped

Tag	Index	Offset
-----	-------	--------

A cache block can only go in one spot in the cache. It makes a cache block very easy to find, but it's not very flexible about where to put the blocks.

2-Way Set Associative

Tag	Index	Offset
-----	-------	--------

This cache is made up of sets that can fit two blocks each. The index is now used to find the set, and the tag helps find the block within the set.

4-Way Set Associative

Tag	Index	Offset
-----	-------	--------

Each set here fits four blocks, so there are fewer sets. As such, fewer index bits are needed.

Fully Associative

Tag	Offset
-----	--------

No index is needed, since a cache block can go anywhere in the cache. Every tag must be compared when finding a block in the cache, but block placement is very flexible!

They all look set associative to me.

That's because they are! The direct mapped cache is just a 1-way set associative cache, and a fully associative cache of m blocks is an m-way set associative cache!

Static WCET analysis /7

- Safeness is at risk
  - When *local* worst case does not always lead to *global* worst case
  - When *timing anomalies* occur
    - Complex hardware architectures (e.g., out-of-order pipelines)
    - Even improper design choices (e.g., cache replacement policies)
    - *Counter-intuitive* timing behavior
    - Faster execution of a single instruction causes *long-term* negative effects
  - Both are very difficult to account for in static analysis

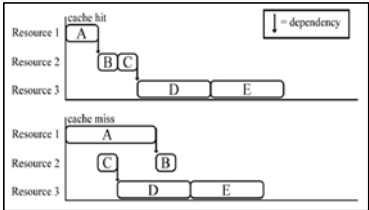
Static WCET analysis: the big picture



- Open problems
  - Can we always trust HW modeling?
  - How much overestimation do we incur?
    - Inclusion of infeasible paths
    - Overestimation intrinsic in abstract state computation
  - Weaknesses of user annotations
    - Labor intensive and error prone

Scheduling anomaly: example

- Some dependence between instructions
- Shared resources (e.g. pipeline stages) and opportunistic scheduling



- Faster execution of A leads to a worse case overall execution because of the order in which instructions are executed

## Hybrid analysis /1

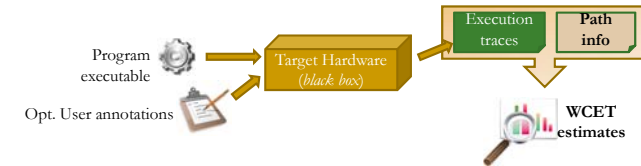
- To obtain *realistic* (less pessimistic) WCET estimates
  - On the real target processor
  - On the final executable
  - Knowing that safeness not guaranteed (!)
- Hybrid approaches exploit
  - The measurement of *basic blocks* on the real HW
    - To avoid pessimism from abstract modeling
  - Static analysis techniques to combine the obtained measures
    - Knowledge of the program execution paths
- Newer approaches explore probabilistic properties (!)

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## Hybrid analysis: the big picture



- Open problems
  - Can we trust the resulting estimates?
    - Contingent on worst-case input and worst-case HW state
    - Consideration of infeasible paths
  - Needs the real execution environment or an identical copy of it
    - May cause serious cost impact and inherent difficulty of exactness

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## Hybrid analysis /2

- Approaches to collect timing information
  - *Software instrumentation*
    - The program is augmented with instrumentation code
    - Instrumentation effects the timing behavior of the program (aka the *probe effect*) and causes problems to deciding what's the final system
  - *Hardware instrumentation*
    - Depends on specialized HW features (e.g., debug interface)
- Confidence in the results contingent on the coverage of the executions and on the exploration of worst-case states
  - Exposed to the same problems as static analysis and measurement
  - *Worst-case state dependence is gone if HW response time is randomized*



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## Summary

- The challenge of computing the WCET
- Static analysis
  - High-level analysis
  - Low-level analysis
- Hybrid analysis (measurement-based)

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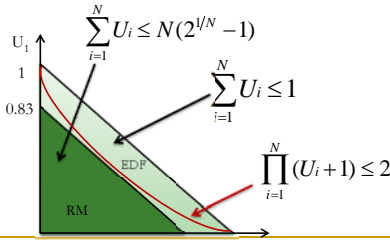
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# 7.b Schedulability analysis techniques

Credits to Marco Panunzio  
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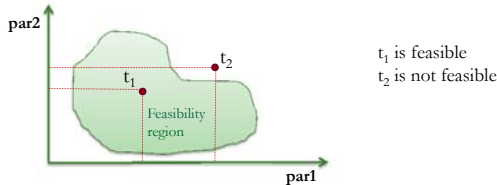
## Advanced utilization tests

- *Hyperbolic bound* improves Liu & Layland utilization test
  - For systems with periodic tasks under FPS and DMPO
  - E. Bini and G. Buttazzo: “*A Hyperbolic Bound for the Rate Monotonic Algorithm*”. Proceedings of the 13<sup>th</sup> ECRTS, 2001



## Feasibility region

- The topological space that represents the set of feasible systems with respect to the workload model parameters
  - N-dimensional space with N-parameter analysis
  - Function of the timing parameters
  - Specific to the scheduling policy in force



## Transactions /1

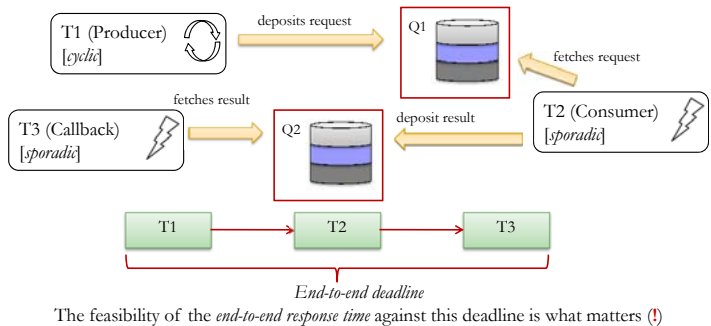
- Causal relations between activities
  - They consider information relevant to analysis that is not captured by classic workload models
    - Dependences in the activation of jobs
- Originally introduced for the analysis of distributed systems
  - Also useful for the analysis of “collaboration patterns” employed for single-CPU systems



Transactions /2

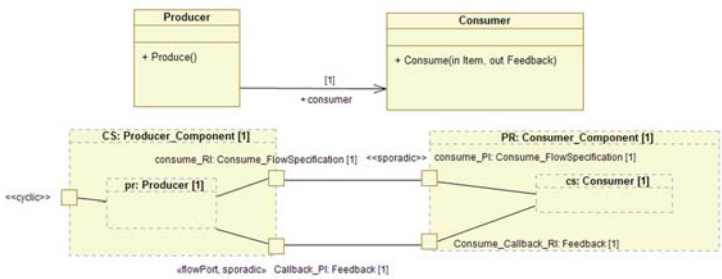
- Two main kinds of dependence
  - Direct precedence** relation (e.g., producer-consumer)
    - $\tau_2$  cannot proceed until  $\tau_1$  completes
  - Indirect priority** relation
    - $\tau_4$  does not suffer interference from  $\tau_3$  (under FPS and synchronous release of  $\tau_1$  and  $\tau_4$  for priorities increasing with values)

Example /2



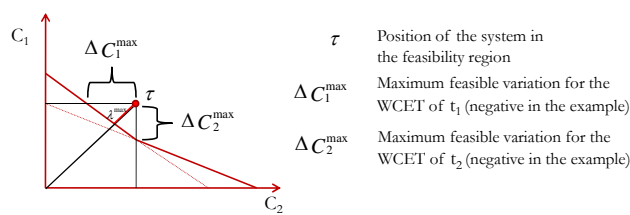
Example /1

- A “callback pattern” to permit **in out** interactions between tasks in Ravenscar systems



Sensitivity analysis /1

- Investigates the changes in a given system that
  - Improve the fit of an already feasible system
  - Make feasible an infeasible system

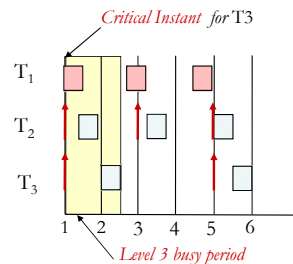


### Sensitivity analysis /2

- Major computation complexity
- Theory still under development
  - Does not account for shared resources, multi-node systems, partitioned systems
- High potential
  - To explore solution space in the *dimensioning* phase of design
    - Presently only applicable to period/MIAT and WCET
  - To study the consequences of changes to timing parameters
    - To allow for the inclusion of better functional value in the system
    - To renegotiate timing (or functional) parameters

### Classic workload model

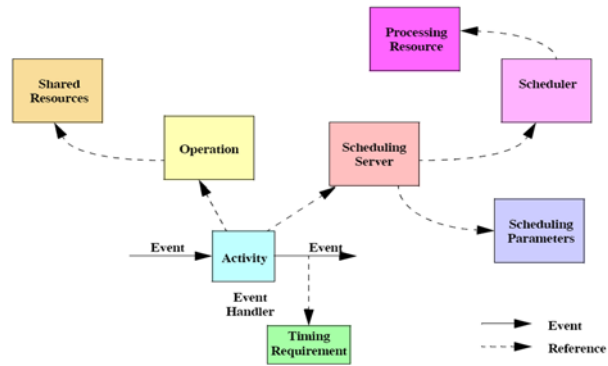
$T_1$ (Sporadic)	MIAT=1.750	WCET=0.500
$T_2$ (Cyclic)	$T=2.000$	WCET=0.500
$T_3$ (Cyclic)	$T=4.000$	WCET=0.500



### MAST

- Modeling and Analysis Suite for Real-Time Systems (MAST, <http://mast.unican.es>)
  - Developed at University of Cantabria, Spain
  - Open source
  - Implements several analysis techniques
    - For uniprocessor and distributed (no-shared memory) processor systems
    - Under FPS or EDF

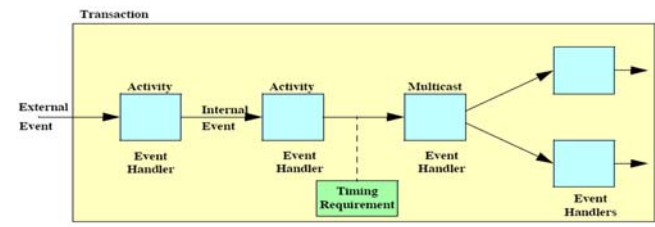
### MAST: real-time model



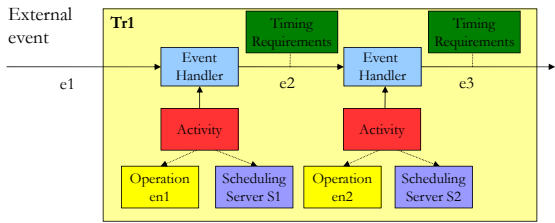


MAST: transaction

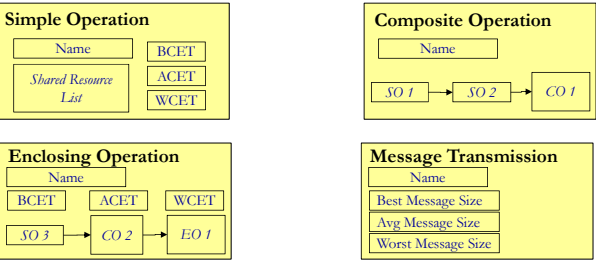
- To model causal relations between activities
  - Triggered by external events
    - Periodic, sporadic, aperiodic, etc...



MAST: creation of a transaction

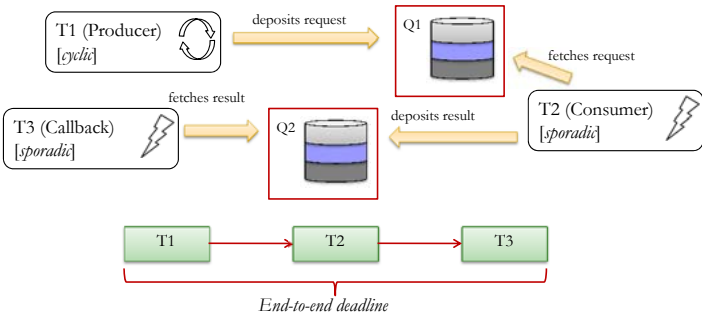


MAST: operations

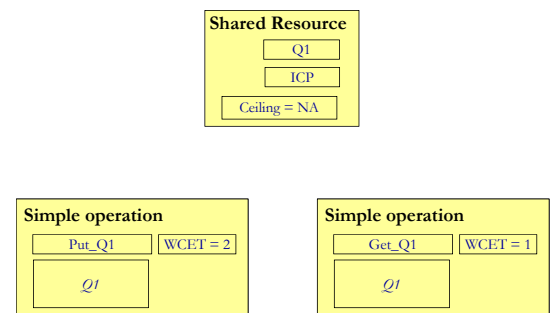


- The real-time model includes the description of all the operations in the system

Example: Ravenscar callback



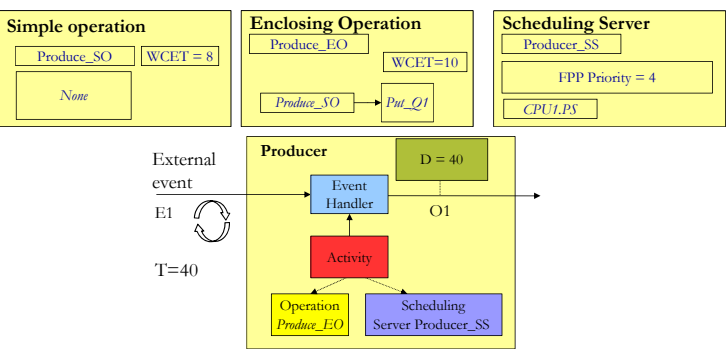
Example: shared resources in MAST



Example: timing attributes

Producer [1]	(C)	$T_1=40$	$C_1=10$	$p_1=4$
Consumer [2]	(S)	$T_2=40$	$C_2=10$	$p_2=2$
Callback [3]	(S)	$T_3=40$	$C_3=5$	$p_3=5$
Q1		Ceiling=4		
Q2		Ceiling=5		

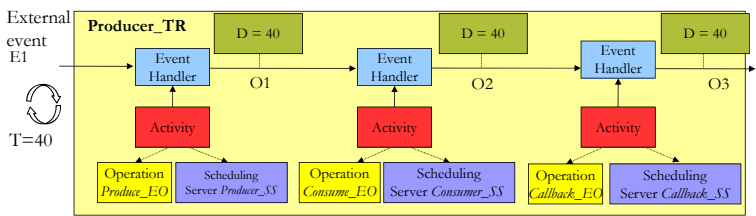
Example: modeling tasks in MAST



Example: classic RTA results

Producer [1]	(C)	$T_1=40$	$C_1=10$	$p_1=4$
Consumer [2]	(S)	$T_2=40$	$C_2=10$	$p_2=2$
Callback [3]	(S)	$T_3=40$	$C_3=5$	$p_3=5$
Q1		Ceiling=4		
Q2		Ceiling=5		
			$B_1=2$	$B_2=0$ $B_3=2$
Classic RTA				
$R_1 = 17$	This misses out completely that $T_3$ is to be <i>preceded</i> by $T_2$ and $T_1$ (!)			
$R_2 = 25$				
$R_3 = 7$				

Example: introducing transactions



Summary

- Feasibility region
- Advanced utilization tests
- Fine-grained response time analysis
- Transactions
- Sensitivity analysis
- Example tool (MAST)

Example: end-to-end analysis

Producer [1]	(C)	$T_1=40$	$C_1=10$	$p_1=4$
Consumer [2]	(S)	$T_2=40$	$C_2=10$	$p_2=2$
Callback [3]	(S)	$T_3=40$	$C_3=5$	$p_3=5$

Q1	Ceiling=4	→	$B_1=2$	$B_2=0$	$B_3=2$
Q2	Ceiling=5				

Classic RTA	Precedence and offset-based	
$R_1 = 17$	$R_1 (Tr) = 12$	
$R_2 = 25$	$R_2 (Tr) = 20$	← Response time relative to the beginning of the transaction!
$R_3 = 7$	$R_3 (Tr) = 27$	