# 6.a Ramifications of schedulability analysis

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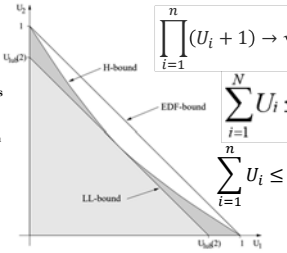
## Example

- *Hyperbolic bound* [Bini & Buttazzo, ECRTS, 2001] improves the Liu & Layland utilization test for RM
  - It helps prove that RM achieves 100% utilization when *all pairs* of periods in the task set are in harmonic relation




**Examples of feasibility regions**
Plot in an $n = 2$ U-space, where each point $U = \{U_1, U_2, \dots, U_n\}$ represents a periodic task set with utilization $U_i$

$$\prod_{i=1}^{n}(U_i + 1) \rightarrow \sqrt{2} + O(n^{-1}) \leq 2$$

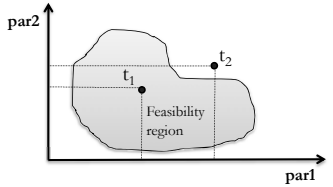
$$\sum_{i=1}^{N} U_i \leq 1$$

$$\sum_{i=1}^{n} U_i \leq n\left(2^{\frac{1}{n}} - 1\right) \rightarrow \leq ln(2)$$



## Feasibility region

- A topological space that represents the set of feasible systems with respect to workload model parameters
  - N-dimensional space for N-parameter analysis
  - Specific to the schedulability tests in use
  - Helps visualize the meaning of feasibility geometrically

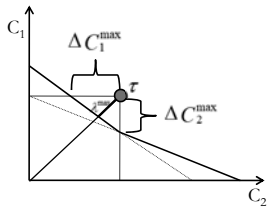



$t_1$ is feasible
$t_2$ is not feasible



## Sensitivity analysis

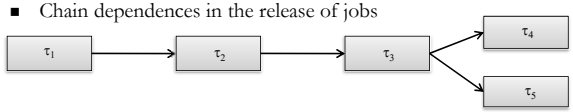
- Investigates the parameter changes in a real-time system that may possibly
  - Improve the goodness of fit of an already feasible system
  - Make feasible an infeasible system




| | |
|---|---|
| $\tau$ | Position of the system in the feasibility region |
| $\Delta C_1^{max}$ | Maximum feasible variation for the WCET of $t_1$ (negative in the example) |
| $\Delta C_2^{max}$ | Maximum feasible variation for the WCET of $t_2$ (negative in the example) |

# Transactions (precedence chains) /1

- Causal relations between activities
  - They allow considering relevant information not captured by classic workload models
    - Chain dependences in the release of jobs



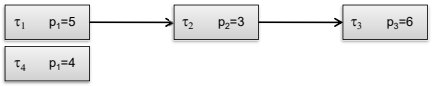

  - Originated in the analysis of distributed systems, where understanding offsets contains the pessimism of release jitter
    - Also useful for the analysis of "*collaboration (release) patterns*" employed for single-CPU systems



# Transactions /2

- Two main kinds of dependence are of interest here
  - *Direct precedence* relation (e.g., producer-consumer)
    - $\tau_2$ cannot proceed until $\tau_1$ completes

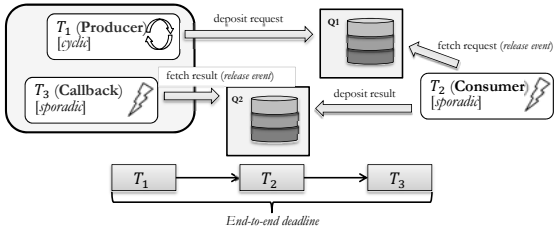

  - *Indirect priority* relation
    - $\tau_4$ does not suffer interference from $\tau_3$ (under FPS and synchronous release of $\tau_1$ and $\tau_4$ for priorities increasing with values)






# Example: Ravenscar call-back

- The "*call-back pattern*" helps realize indirect **in-out** interactions between tasks in Ravenscar systems




- The feasibility of the *end-to-end response time* against the corresponding deadline is the matter of interest here (**!**)



# Example: classic RTA results

| Id | Task | $T_i$ | $C_i$ | Priority | Blocking |
|---|---|---|---|---|---|
| $\tau_1$ | Producer (periodic) | 40 | 10 | 4 | $B_1 = 2$ |
| $\tau_2$ | Consumer (sporadic) | 40 | 10 | 2 (L) | $B_2 = 0$ |
| $\tau_3$ | Call-back (sporadic) | 40 | 5 | 5 (H) | $B_3 = 2$ |

**Q1** Ceiling $= max(P_1, P_2) = 4$

**Q2** Ceiling $= max(P_2, P_3) = 5$

**Classic RTA**

$R_1 = 17$
$R_2 = 25$
$R_3 = 7$

This misses out completely that $\tau_3$ is to be *preceded* by $\tau_2$ and $\tau_1$ (!)

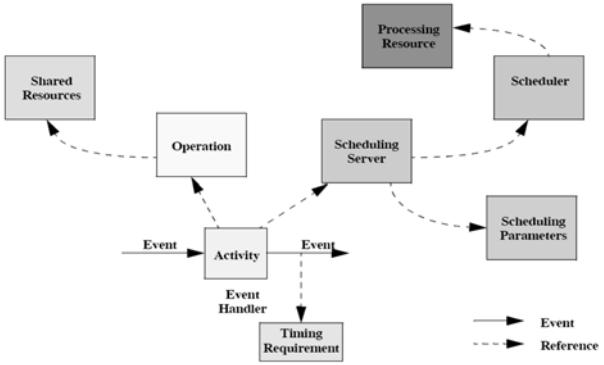
$$R_i = C_i + B_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

# MAST

- Modeling and Analysis Suite for Real-Time Systems (MAST, http://mast.unican.es)
  - Developed at University of Cantabria, Spain
  - Open source
  - Implements several analysis techniques
    - For uniprocessor and distributed (no-shared memory) processor systems
    - Under FPS or EDF



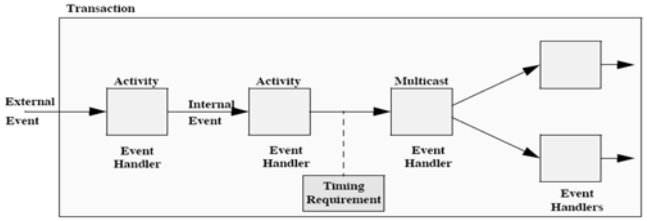
# MAST: real-time model





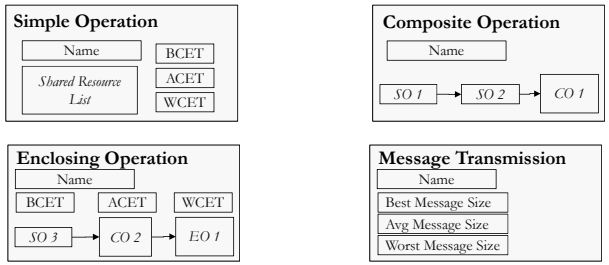

# MAST: transaction

- To model causal relations between activities
  - Triggered by external events
    - Periodic, sporadic, aperiodic, etc…



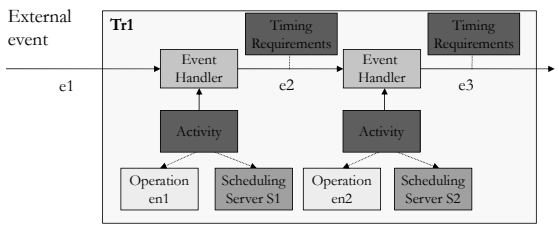



# MAST: operations



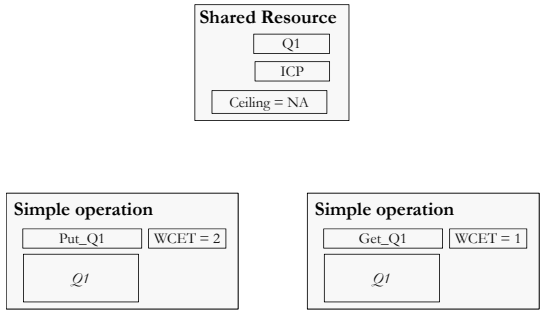

- The real-time model includes the description of all the operations in the system

# MAST: an example transaction



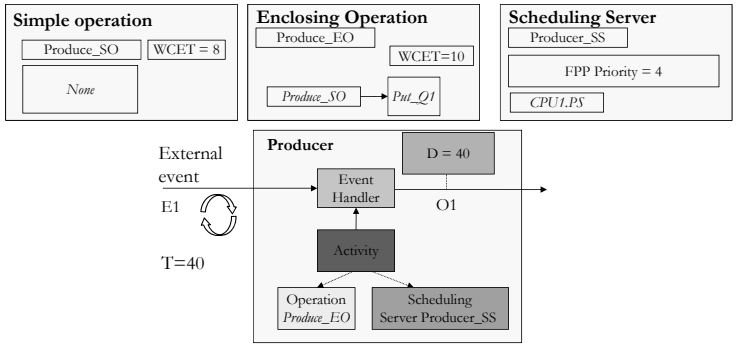



# MAST: example shared resources



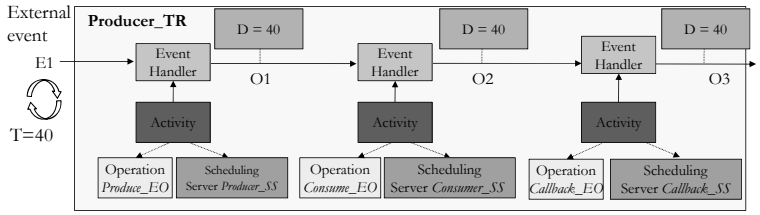



# MAST: example tasks






# Example: introducing transactions



# Example: end-to-end analysis

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**Classic RTA**

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$R_3 = 7$

**Precedence and offset-based RTA**

$R_1 = 12$

$R_2 = 20, O_2 = R_1^{best}, J_2 = R_1 - R_1^{best}$

$R_3 = 27$ ⇐ **Relative to the beginning of the transaction, not knowing the best case**

$$R_i = C_i + B_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i - O_j + J_j + O_i + J_i}{T_j} \right\rceil C_j - O_i + J_i$$



# Summary

- Feasibility region
- Advanced utilization tests
- Sensitivity analysis
- Transactions
- Example with MAST