3.a Fixed-Priority Scheduling

Credits to A. Burns and A. Wellings

The simplest workload model

- The application consists of n tasks, for constant n
- All tasks are *periodic* with known periods
  - This defines the *periodic workload model*
- All tasks are completely *independent* of each other
  - No contention for logical resources; no precedence constraints
- All tasks have implicit deadline $(D = T)$
  - Each job of task must complete before the next job is released
- All tasks have a single fixed WCET, which can be trusted as *a safe and tight upper-bound*
- All system overheads (context-switch times, interrupt handling and so on) are assumed absorbed in the WCETs

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Notation in this section

B:	Worst-case blocking time for the task (if applicable)
C:	Worst-case computation time (WCET) of the task ($= e$)
D:	Relative deadline of the task
I:	The interference time of the task
J:	Release jitter of the task
N:	Number of tasks in the system
P:	Priority assigned to the task (if applicable)
R:	Worst-case response time of the task
T:	Minimum time between task releases, or task period ($= p$)
U:	The utilization of each task ($= {}^{C}/_{T}$)
a-Z:	The name of a task

Fixed-priority scheduling (FPS)

- Still the most widely used approach in industry
- Each task has a fixed (static) priority determined off-line
- In real-time systems, the "priority" of a task is solely derived from its temporal requirements
  - The task's relative importance (criticality) to the correct functioning of the system or its integrity is not a factor at this level
  - A recent strand of research addresses **mixed-criticality systems**, with scheduling solutions that contemplate *criticality* attributes

- The ready jobs are dispatched to execution in the order determined by their (static) priority
- In FPS, scheduling at run time is fully defined by the (static) priority assignment algorithm

Preemption and non-preemption /1

- With priority-based scheduling, a high-priority task may be released during the execution of a lower priority one
- In a *preemptive* scheme, there will be an immediate switch to the higher-priority task
- With *non-preemption*, the lower-priority task will be allowed to complete before the one notionally at the top of the ready queue may execute
- Preemptive schemes (such as FPS and EDF) enable higher-priority tasks to be more reactive, hence they are preferred

Preemption and non-preemption /2

- Alternative strategies allow a lower priority task to continue executing for a bounded time before being preempted
- Such schemes use either *deferred preemption* or *cooperative dispatching*
- ***Value-based scheduling*** (VBS) is another approach to attenuating preemption
  - Useful when the system becomes overloaded and some adaptive scheme of scheduling is needed to mitigate the risk or the consequences of overrun
  - VBS assigns a value to each task and then employs an on-line value-based scheduling algorithm to decide which task to run next
  - Analogous to usefulness, but determined off-line

Rate-monotonic priority assignment

- Each task is assigned a priority based on its period
  - The shorter the period, the higher the priority
  - Such priorities have to be unique: hence ties must be resolved
- For any two tasks $\tau_i, \tau_j : T_i < T_j \rightarrow P_i > P_j$
  - ***Rate monotonic*** assignment is **optimal** under preemptive priority-based scheduling (and implicit deadlines)
- **Nomenclature**
  - Priority 1 as numerical value is the lowest (least) priority
  - However, the task indices are sorted highest to lowest

Utilization-based test

- A simple test exists for rate-monotonic scheduling
- It provides a *sufficient but not necessary* upper-bound on the schedulable utilization of FPS
  - Only for task sets with $D = T$

$$U = \sum_{i=1}^{n} \frac{C_i}{T_i} \leq n\left(2^{\frac{1}{n}} - 1\right)$$

$$\lim_{n \to \infty} n\left(2^{\frac{1}{n}} - 1\right) = \ln 2 = 0.69$$

- The schedulable utilization of FPS is *less* than EDF

Critique of utilization-based tests

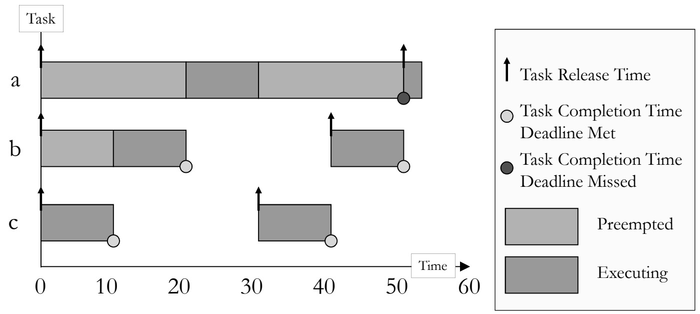
- These tests are sufficient but not necessary
  - As such, they fall in the class of *schedulability tests*
- These tests are not exact and also not general
- But they are $\Omega(n)$, which makes them interesting for some users

Example: task set A

Task	Period	Computation Time	Priority	Utilization
	T	**C**	**P**	**U**
a	50	12	1 (low)	0.24
b	40	10	2	0.25
c	30	10	3 (high)	0.33

- The combined utilization is 0.82 (or 82%)
- Above the threshold for three tasks (0.78)
  - This task set fails the utilization test
- Hence we have no a-priori answer on its feasibility

Timeline for task set A



Example: task set B

Task	Period	Computation Time	Priority	Utilization
	T	**C**	**P**	**U**
a	80	32	1 (low)	0.40
b	40	5	2	0.125
c	16	4	3 (high)	0.25

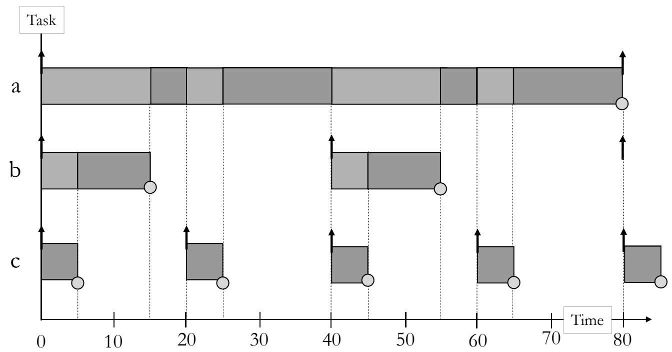
- The combined utilization is 0.775
- Below the threshold for three tasks (0.78)
  - This task set passes the utilization test
- Hence this task set will meet all its deadlines

Example: task set C

Task	Period	Computation Time	Priority	Utilization
	T	C	P	U
a	80	40	1 (low)	0.50
b	40	10	2	0.25
c	20	5	3 (high)	0.25

- The combined utilization is 1.0
- Above the threshold for three tasks (0.78)
  - Again, this task set does not pass the utilization test
- Yet the timeline shows the task set will meet all its deadlines

Timeline for task set C



Response time analysis /1

- The worst-case response time R_i of task τ_i is first calculated and then checked (trivially) with its deadline
- τ_i is feasible if and only if $R_i \le D_i$
- $R_i = C_i + I_i$, where I_i is the *interference* that τ_i suffers from higher-priority tasks

Calculating R

- Within R_i, each higher priority task τ_j will execute at most $\left\lceil \frac{R_i}{T_j} \right\rceil$ times
  - The ceiling function $\lceil f \rceil$ gives the smallest integer greater than the fractional number f on which it acts
    - E.g., the ceiling of 1/3 is 1, of 6/5 is 2, and of 6/3 is 2
  - Using the ceiling reflects the fact that τ_i will be preempted for a *full* execution of a higher-priority released exactly at τ_i's end
- The total interference suffered by τ_i from τ_j in R_i where $P_i < P_j$, is given by $\left\lceil \frac{R_i}{T_j} \right\rceil C_j$

Response time equation

$$R_i = C_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

- Where $hp(i)$ is the set of tasks with priority higher than τ_i
- Solved by forming a recurrence relationship

$$w_i^{n+1} = C_i + \sum_{j \in hp(i)} \left\lceil \frac{w_i^n}{T_j} \right\rceil C_j$$

- The set of values $w_i^0, w_i^1, w_i^2, \ldots, w_i^n, \ldots$ is monotonically non-decreasing
- When $w_i^n = w_i^{n+1}$ the solution to the equation has been found
- w_i^0 must not be greater than C_i (e.g. 0 or C_i)

Response time algorithm

```
for i in 1..N loop -- for each task in turn
  n := 0
  w_i^n := C_i
loop
    calculate new w_i^{n+1}
    if w_i^{n+1} = w_i^n then
      R_i = w_i^n
      exit value found
    end if
    if w_i^{n+1} > T_i then
      exit value not found
    end if
    n := n + 1
  end loop
end loop
```

If the recurrence does not converge before T_i we can still set a termination condition that attempts to determine how long past T_i, job i completes

Example: task set D

Task	Period	Computation Time	Priority	Utilization
	T	C	P	U
a	7	3	3 (high)	0.4285…
b	12	3	2	0.25
c	20	5	1 (low)	0.25

$R_a = 3$

$$w_b^0 = 3$$
$$w_b^1 = 3 + \left\lceil \frac{3}{7} \right\rceil 3 = 6$$
$$w_b^2 = 3 + \left\lceil \frac{6}{7} \right\rceil 3 = 6$$

$R_b = 6$

Example (cont'd)

$$w_c^0 = 5$$
$$w_c^1 = 5 + \left\lceil \frac{5}{7} \right\rceil 3 + \left\lceil \frac{5}{12} \right\rceil 3 = 11$$
$$w_c^2 = 5 + \left\lceil \frac{11}{7} \right\rceil 3 + \left\lceil \frac{11}{12} \right\rceil 3 = 14$$
$$w_c^3 = 5 + \left\lceil \frac{14}{7} \right\rceil 3 + \left\lceil \frac{14}{12} \right\rceil 3 = 17$$
$$w_c^4 = 5 + \left\lceil \frac{17}{7} \right\rceil 3 + \left\lceil \frac{17}{12} \right\rceil 3 = 20$$
$$w_c^5 = 5 + \left\lceil \frac{20}{7} \right\rceil 3 + \left\lceil \frac{20}{12} \right\rceil 3 = 20$$

$R_c = 20$

Revisiting task set C

Task	Period	Computation Time	Priority	Response Time
	T	C	P	R
a	80	40	1 (low)	80
b	40	10	2	15
c	20	5	3 (high)	5

- The combined utilization is 1.0, above the utilization threshold for three tasks (0.78)
  - Hence the utilization test fails
- But RTA shows that the task set will meet all its deadlines
  - Cf. the impasse we had at pages 178-179

Response time analysis /2

- RTA is a *feasibility test*
  - Exact, hence necessary and sufficient
- If the task set passes the test then all its tasks will meet all their deadlines
- If it fails the test then, at run time, some tasks will miss their deadline and FPS tells us exactly which
  - Unless the computation time estimations (the WCET) themselves turn out to be pessimistic

Sporadic tasks

- Sporadic tasks have a ***minimum inter-arrival time***
  - This should be preserved at run time if schedulability is to be ensured, but how can it **?**
- The RTA for FPS works perfectly well for $D \leq T$ as long as the stopping criterion becomes $W_i^{n+1} > D_i$
- Interestingly, RTA also works perfectly well with *any* priority ordering, as long as the task indices reflect it

Hard and soft tasks

- In many situations the WCET given for sporadic tasks are considerably higher than the average case
- Interrupts often arrive in bursts and an abnormal sensor reading may lead to significant additional computation
- Measuring feasibility with WCET may lead to very low processor utilization being frequently observed at run time
  - We need some common sense to contain pessimism

General common-sense guidelines

- **Rule 1** : All tasks (hard and soft) should be schedulable using *average* execution times and average arrival rates for both periodic and sporadic tasks
  - There may therefore be situations in which it is not possible to meet all current deadlines
  - This condition is known as a *transient overload*
- **Rule 2** : All hard real-time tasks should be schedulable using WCET and worst-case arrival rates of all tasks (including soft)
  - No hard real-time task will therefore miss its deadline
  - If Rule 2 incurs unacceptably low utilizations for non-worst-case jobs then WCET values or arrival rates must be reduced

Handing aperiodic tasks /1

- They do *not* have minimum inter-arrival times
  - And consequently no deadline
  - We may be interested in the system being responsive to them (in cyclic scheduling we would use *slack stealing* for them)
- We can run aperiodic tasks at a priority below the priorities assigned to hard tasks
  - In a preemptive system, they won't steal resources from hard tasks
- But this does not provide adequate support to soft tasks which would often miss their deadlines
- We need another kind of solution …

Handing aperiodic tasks /2

- In addition to preserving hard tasks and giving fair opportunities to soft tasks, we need a solution that minimizes
  - The response time of the job *at the head* of the aperiodic queue
  - Or the average response time of *as many* aperiodic jobs as possible for a given queuing discipline
- Possible solutions
  - Execute the aperiodic jobs in the background
  - Execute the aperiodic jobs by interrupting the periodic jobs
  - Use slack stealing

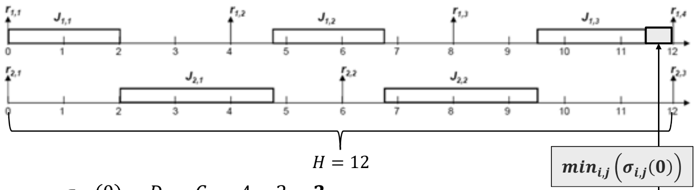
  - Use dedicated servers

Handing aperiodic tasks /3

- ***Slack stealing***
  - Difficult to implement for preemptive systems
    - The slack $\sigma(t)$ is a *not* a constant for them
    - It is a function of the time t at which it is computed
  - The slack stealer is ready when the aperiodic queue is not empty; it is suspended otherwise
  - When ready and $\sigma(t) > 0$, the slack stealer is assigned the highest priority; the lowest when $\sigma(t) = 0$
  - Static computation of $\sigma(t)$ for some t is useful but only when the release jitter in the system is very low
    - Under EDF, $\sigma(t = 0) = min_i\{\sigma_i(0)\}$ where $\sigma_i(0) = D_i - \sum_{k=1,..,i} e_k$ for *all* jobs released in the hyperperiod starting at $t = 0$

Computing the slack under EDF

$T_1 = (4, 2)$, $T_2 = (6, 2.75)$ - EDF scheduling: (~~ϕ_i~~, p_i, e_i, ~~D_i~~)



$\sigma_{1,1}(0) = D_1 - C_1 = 4 - 2 = \mathbf{2}$

$\sigma_{2,1}(0) = D_2 - C_1 - C_2 = 6 - 2 - 2.75 = \mathbf{1.25}$

$\sigma_{1,2}(0) = D_{1_2} - 2 \times C_1 - C_2 = 8 - 2 \times 2 - 2.75 = \mathbf{1.25}$

$\sigma_{2,2}(0) = D_{2_2} - 2 \times C_1 - 2 \times C_2 = 12 - 2 \times 2 - 2 \times 2.75 = \mathbf{2.5}$

$\sigma_{1,3}(0) = D_{1_3} - 3 \times C_1 - 2 \times C_2 = 12 - 3 \times 2 - 2 \times 2.75 = \mathbf{0.5}$

Computing the slack under FPS /1

- The amount of slack that an FPS system has in a time interval may depend on *when* the slack is used
- To minimise the response time of an aperiodic job J_a the decision on when to schedule it must consider the execution time of J_a
  - *No* slack stealing algorithm under FPS can minimise the response time of *every* aperiodic job even with prior knowledge of their arrival and execution times
  - Better *not* be greedy in using the available slack

Computing the slack under FPS /2

- The slack of periodic jobs of τ_i should be computed based on their *effective deadline* D_i^e
  - For a job of τ_i, it should be computed at the beginning of the level-$i-1$ busy period that precedes D_i so that $D_i^e \leq D_i$
- The initial slack $\sigma_{i,j}(0)$ of every periodic job J_{ij} (the j^{th} job of task J_i) in H is determined as

$$max\left(0, D_{ij}^e - \sum_{k=1}^{i} \left\lceil \frac{D_{ij}^e}{T_k} \right\rceil C_k\right)$$

Slack stealing defeats optimality

- Greed is no good: to minimize the response time of an aperiodic job, it may be necessary to schedule it later, even if slack is currently available
  - For any periodic task set, under any FPS, and any aperiodic queuing policy, *no* valid algorithm exists that minimizes the response time of *all* aperiodic jobs
  - Similarly, no valid algorithm exists that minimizes the average response time of the aperiodic jobs

T.-S. Tia, J. W.-S. Liu, and M. Shankar, "Algorithms and Optimality of Scheduling Aperiodic Requests in Fixed-Priority Preemptive Systems," Journal of Real-Time Systems, 10(1), pp. 23-43, 1996.

Handing aperiodic tasks /4

- ***Periodic server*** (PS) – general model
  - A notional (T_{ps}, C_{ps}) periodic task scheduled at the highest priority to only execute aperiodic jobs
    - The PS has a ***budget*** C_{ps} time units and a ***replenishment period*** of length T_{ps}
    - When the PS is scheduled and executes aperiodic jobs, it consumes its budget at the rate of 1 unit per unit of time
    - Budget is exhausted when $C_{ps} = 0$ and replenished periodically
  - The PS is *backlogged* when the aperiodic job queue is nonempty and it is idle otherwise
    - Eligible for execution only when ready, backlogged and $C_{ps} > 0$

Handing aperiodic tasks /5

- ***Polling server***, a simple (naïve) kind of PS
  - It is given a fixed budget that it uses to serve aperiodic task requests that is replenished at every period
  - The budget is immediately consumed if the server is scheduled while idle
  - It is *not* ***bandwidth preserving***, hence inefficient
    - An aperiodic job that arrives just after the server has been scheduled while idle, must wait until the next replenishment time
  - Bandwidth-preserving servers need additional rules for consumption and replenishment of their budget

Handing aperiodic tasks /6

- ***Deferrable Server*** (DS), a *bandwidth-preserving* PS
  - DS retains its budget if no aperiodic tasks require execution
    - If an aperiodic job requires execution during the DS period, it can be served immediately: when idle, the DS stays ready (not idle)
  - The budget is replenished at the start of the new period (!)
    - If an aperiodic job arrives ε time units before the end of T_{ds}, the request begins to be served and blocks periodic tasks
    - When the budget is replenished, new aperiodic jobs may then be served for the full budget
  - If that happens, in $\omega(t)$, DS contributes a solid interference of $C_{ds} + \left\lceil \frac{t-C_{ds}}{T_{ds}} \right\rceil C_{ds}$, *longer* than $1 \times C_{ds}$ per busy period

Handing aperiodic tasks /7

- ***Priority Exchange*** (PE), similar in principle to DS
  - If PE is idle when scheduled, it exchanges its own priority with that of the pending periodic task with priority lower than itself and higher of all other pending periodic tasks
  - The selected periodic task <u>inherits</u> PE's higher priority until an aperiodic task arrives or PE's ready period ends

Handing aperiodic tasks /8

- ***Sporadic Server*** (SS), fixes the bug in DS
  - The budget is replenished only when exhausted and at a minimum guaranteed distance from its earlier execution
    - Hence no longer at a fixed rate
  - This places a tighter bound on its interference and makes schedulability analysis simpler and less pessimistic
- This is the default server policy in POSIX

SS rules under FPS

- ***Consumption rules***
  - At time $t > t_r$ (the latest replenishment time), a backlogged SS consumes budget only if executing, hence when no higher-priority task is ready
  - The replenishment is limited to the quantity of actual consumption
- ***Replenishment rules***
  - t_r records the time that SS' budget was last replenished
  - t_e records the time when SS first begins to execute since t_r
    - $t_e > t_r$ is the latest time at which a lower-priority task than SS executes
  - The next replenishment time is set to $t_e + T_{ss}$
- ***Exception***
  - If only higher-priority tasks had been busy since t_r, then $t_e + T_{ss} > t_r + T_{SS}$ and SS is late: hence, budget fully replenished as soon as exhausted

SS rules unveiled

- Let t_a be the time at which SS has full budget *and* becomes backlogged, and $t_f \geq t_a$ the time at which SS becomes idle
- In the $[t_a, t_f]$ interval, when SS is continuously active, three cases are possible

1. SS has consumed no capacity: $t_{r_{next}} = t_f + T_{SS}$ → no replenishment, and no interference in that interval
2. SS has consumed all capacity: $t_{r_{next}} = t_a + T_{SS}$ → full replenishment, and bounded interference in that interval
3. SS has consumed fractional capacity: $t_{r_{next}} = t_f + T_{SS}$ → fractional replenishment, and interference lower than allowed in that interval

Handing aperiodic tasks /9

- SS is more complex than PS or DS
  - Its rules require keeping tab of lots of data
  - Several cases to consider when making scheduling decisions
  - This complexity is acceptable because the schedulability of a SS is easy to demonstrate
    - Under FPS, SS equates to a periodic task τ_s with (p_s, e_s)
- EDF and LLF use a dynamic variant of SS as well as other bandwidth-preserving server algorithms known as
  - *Constant utilization server*
  - *Total bandwidth server*
  - *Weighted fair queuing server*

Task sets with D < T

- For $D = T$, Rate Monotonic priority assignment (a.k.a. ordering) is optimal
- For $D < T$, ***Deadline Monotonic*** priority ordering is optimal

$$D_i < D_j \Rightarrow P_i > P_j$$

DMPO is optimal /1

- Deadline monotonic priority ordering (**DMPO**) is optimal

  any task set Q that is schedulable by priority-driven scheme W it is also schedulable by DMPO

- The proof of optimality of DMPO involves transforming the priorities of Q as assigned by W until the ordering becomes as assigned by DMPO
- Each step of the transformation will preserve schedulability

DMPO is optimal /2

- Let τ_i, τ_j be two tasks with adjacent priorities in Q such that under W we have $P_i > P_j \land D_i > D_j$
- Define scheme W' to be identical to W except that tasks τ_i, τ_j are swapped
- Now consider the schedulability of Q under W'
- All tasks $\{\tau_k\}$ with priority $P_k > P_j$ will be unaffected
- All tasks $\{\tau_s\}$ with priority $P_s < P_i$ will be unaffected as they will experience the same interference from τ_j and τ_i
- Task τ_j which was schedulable under W, now has a higher priority, suffers less interference, and hence must be schedulable under W'

DMPO is optimal /3

- All that is left to show is that task τ_i, which has had its priority lowered, is still schedulable
- Under W we have $R_j \leq D_j, D_j < D_i$ and $R_i \leq T_i$
- Task τ_j only interferes once during the execution of task τ_i hence $R_i' = R_j \leq D_j < D_i$
  - Under W' task τ_i completes at the time task τ_j did under W
  - Hence task τ_i is still schedulable after the switch
- Priority scheme W' can now be transformed to W'' by choosing two more tasks that are in the wrong order for DMPO and switching them

Summary

- A simple (periodic) workload model
- Delving into fixed-priority scheduling
- A (rapid) survey of schedulability tests
- Some extensions to the workload model
- Priority assignment techniques

Selected readings

- N.C. Audsley, A. Burns, R.I. Davis, K.W. Tindell, A.J. Wellings (**1995**)
  Fixed priority pre-emptive scheduling: an historical perspective
  DOI: 10.1007/BF01094342
- D. Faggioli, M. Bertogna, F. Checconi (**2010**)
  Sporadic Server revisited
  DOI: 10.1145/1774088.1774160