3.a Fixed-Priority Scheduling

Credits to A. Burns and A. Wellings

Where we look at the schedulability tests for FPS, their strength and weaknesses, we accommodate aperiodic tasks, and we review the priority assignment algorithms

The simplest workload model

- The application consists of n tasks, for constant n
- All tasks are *periodic* with known periods
  - Whence the name "*periodic workload model*"
- All tasks are assumed *independent*
  - No sharing of logical resources; no precedence constraints
- All tasks have implicit deadline $(D = T)$
  - Each job must complete before the release of its successor
- All tasks have a single, fixed WCET
  - Which can be trusted as *a safe and tight upper-bound*
- All runtime overheads are collated in the tasks' WCET
  - Context-switch times, handing of clock interrupts, etc.

Notation in this section

B:	Worst-case blocking time for the task (if applicable)
C:	Worst-case computation time (WCET) of the task (= e)
D:	Relative deadline of the task
I:	The interference time of the task
J:	Release jitter of the task
N:	Number of tasks in the system
P:	Priority assigned to the task (if applicable)
R:	Worst-case response time of the task
T:	Minimum time between task releases, or task period (= p)
U:	The utilization of each task (= C/T)
a-Z:	The name of a task

Fixed-priority scheduling (FPS)

- Still the most widely used approach in industry
- Each task has a fixed (static) priority determined off-line
- The "priority" of a real-time task is solely derived from its temporal requirements
  - The task's relative importance (aka criticality) to correct system operation or system integrity does *not* influence its scheduling priority
  - Later in this course we shall discuss **mixed-criticality systems**, which employ scheduling solutions that also contemplate *criticality* attributes

- The ready jobs are dispatched to execution in the order determined by the static priority of their corresponding task
  - FPS at run time if fully determined by the priority assignment algorithm!

Preemption and non-preemption /1

- With priority-based scheduling, a high-priority task may released a job during the execution of a lower-priority one
  - The HP job will be placed at the top of the (notional) ready queue
- In a *preemptive* scheme, that event will cause an immediate switch of execution to the HP job
- With *non-preemption*, the LP job will be allowed to complete before the job at the top of the ready queue may execute
- Preemptive schemes (such as FPS and EDF) enable higher-priority tasks to be more reactive, hence they are preferred
  - Non-preemptive scheme protect "delicate" fractions of execution

Preemption and non-preemption /2

- Alternative strategies allow a LP job to continue executing for a bounded time before being preempted
  - Earlier than its completion
- Such schemes use either *deferred preemption* ("give me a little bit more") or *cooperative dispatching* ("I will tell you when")
- ***Value-based scheduling*** (VBS) is another way to control preemption
  - When the system becomes overloaded, some adaptive scheme of scheduling helps mitigate the risk or the consequences of overrun
  - A utility value is attached to each task off-line, and an on-line VBS algorithm to decide which job to run next

Rate-monotonic scheduling (RMS)

- Each task is assigned a priority based on its period
  - The shorter the period, the higher the priority
  - Such priorities have to be unique: no ties allowed
- For any two tasks $\tau_i, \tau_j : T_i < T_j \rightarrow P_i > P_j$
  - ***Rate monotonic*** assignment is **optimal** under preemptive priority-based scheduling and implicit deadlines
- **Oddity of nomenclature**
  - Priority 1 as numerical value is the lowest (least) priority
  - Task indices are sorted highest-priority to lowest-priority

Utilization-based test

- A simple *sufficient but not necessary* test exists for RMS for task sets with $D = T$
  - It upper-bounds the schedulable utilization of FPS

$$U(n) = \sum_{i=1}^{n} \frac{C_i}{T_i} \leq n\left(2^{\frac{1}{n}} - 1\right)$$

where $\lim_{n\rightarrow\infty} n\left(2^{\frac{1}{n}} - 1\right) = \ln 2 \sim 0.69$

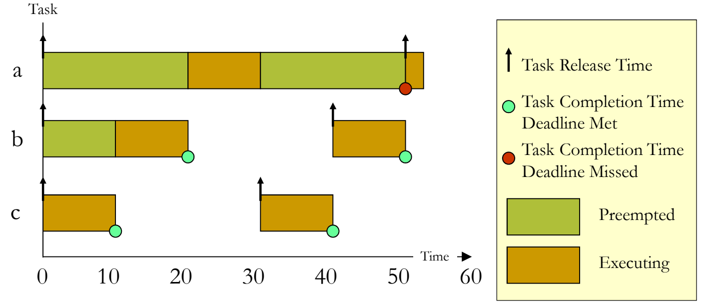
- This shows that the schedulable utilization of FPS (RMS) is *less* than that of EDF
- Utilization-based tests are simple to compute, but highly inaccurate: they often *don't know* …

Example: task set A

Task	Period	Computation Time	Priority	Utilization
	T	C	P	U
a	50	12	1 (low)	0.24
b	40	10	2	0.25
c	30	10	3 (high)	0.33

- The combined utilization of this task set is $U_A = 0.82$
- Above the threshold for three tasks: $U_A > U(3) = 0.78$
  - Task set A fails the utilization-based test
- Hence, we have no a-priori answer on its actual feasibility

Timeline for task set A



Example: task set B

Task	Period	Computation Time	Priority	Utilization
	T	C	P	U
a	80	32	1 (low)	0.40
b	40	5	2	0.125
c	16	4	3 (high)	0.25

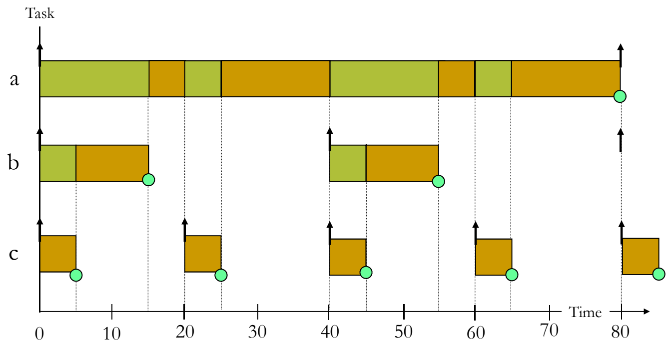
- Its combined utilization is $U_B = 0.775 < U(3) = 0.78$
  - It passes the utilization-based test
- Hence, this task set is guaranteed to meet all its deadlines

Example: task set C

Task	Period	Computation Time	Priority	Utilization
	T	C	P	U
a	80	40	1 (low)	0.50
b	40	10	2	0.25
c	20	5	3 (high)	0.25

- Its combined utilization is $U_C = 1.0 > U(3) = 0.78$
  - It fails the utilization-based test
  - But, interestingly, the task periods are harmonic
- The timeline shows that the task set meets all its deadlines
  - FPS (RMS) performs much better with harmonic-rate tasks

Timeline for task set C



Response time analysis /1

- RTA is a *feasibility test* : it is exact, hence necessary and sufficient
  - If the task set passes the test, then all its tasks will meet all their deadlines
  - If it fails the test, then some tasks will miss their deadline at run time
    - Unless the WCET values turn out to be pessimistic
- FPS determines exactly which tasks will miss their deadline in that case

Response time analysis /2

- The worst-case response time R_i of task τ_i is first calculated and then checked with its deadline D_i
  - τ_i is feasible if and only if $R_i \leq D_i$
- $R_i = C_i + I_i$, where I_i denotes the *interference* that τ_i suffers from higher-priority tasks
- With feasibility analysis we reason about tasks, but scheduling applies to their jobs!

Calculating R

- Within the span of R_i, each HP task τ_j will execute at most $\left\lceil \frac{R_i}{T_j} \right\rceil$ times
  - The ceiling function $\lceil f \rceil$ gives the smallest integer greater than the fractional number f on which it acts
    - E.g., the ceiling of 1/3 is 1, of 6/5 is 2, as it is of 6/3
  - Using the ceiling signifies that a job of τ_i will be preempted for a *full* execution of a job of τ_j released *exactly* at τ_i's end
- The total interference suffered by τ_i from τ_j in R_i where $P_i < P_j$, is upper-bounded by $\left\lceil \frac{R_i}{T_j} \right\rceil C_j$

Response time equation

$$R_i = C_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

- Where $hp(i)$ is the set of tasks with priority higher than τ_i's
- Solved by forming a recurrence relation

$$w_i^{n+1} = C_i + \sum_{j \in hp(i)} \left\lceil \frac{w_i^n}{T_j} \right\rceil C_j$$

- The set of values $w_i^0, w_i^1, w_i^2, \ldots, w_i^n$ is *monotonically non-decreasing*
  - w_i^0 must not be greater than C_i besides being non-negative
- When $w_i^n = w_i^{n+1}$, the solution to the equation has been found

Response time algorithm

```
for i in 1..N loop -- for each task in turn
  n := 0
  w_i^n := C_i
loop
    calculate new w_i^(n+1)
    if  w_i^(n+1) = w_i^n  then
      w_i^(n+1)
      exit value found
    end if
    if  w_i^(n+1) > T_i  then
      exit value not found
    end if
    n := n + 1
  end loop
end loop
```

If the recurrence does not converge before T_i we can still set a termination condition that attempts to determine how long past T_i, job i completes

Example: task set D

Task	Period	Computation Time	Priority	Utilization
	T	C	P	U
a	7	3	3 (high)	0.4285…
b	12	3	2	0.25
c	20	5	1 (low)	0.25

$$\boxed{R_a = 3}$$

$$\begin{cases} w_b^0 = 3 \\ w_b^1 = 3 + \left\lceil \frac{3}{7} \right\rceil 3 = 6 \\ w_b^2 = 3 + \left\lceil \frac{6}{7} \right\rceil 3 = 6 \\ \boxed{R_b = 6} \end{cases}$$

Example (cont'd)

$$\begin{cases} w_c^0 = 5 \\ w_c^1 = 5 + \left\lceil \frac{5}{7} \right\rceil 3 + \left\lceil \frac{5}{12} \right\rceil 3 = 11 \\ w_c^2 = 5 + \left\lceil \frac{11}{7} \right\rceil 3 + \left\lceil \frac{11}{12} \right\rceil 3 = 14 \\ w_c^3 = 5 + \left\lceil \frac{14}{7} \right\rceil 3 + \left\lceil \frac{14}{12} \right\rceil 3 = 17 \\ w_c^4 = 5 + \left\lceil \frac{17}{7} \right\rceil 3 + \left\lceil \frac{17}{12} \right\rceil 3 = 20 \\ w_c^5 = 5 + \left\lceil \frac{20}{7} \right\rceil 3 + \left\lceil \frac{20}{12} \right\rceil 3 = 20 \\ \boxed{R_c = 20} \end{cases}$$

Revisiting task set C

Task	Period	Computation Time	Priority	Response Time
	T	C	P	R
a	80	40	1 (low)	80
b	40	10	2	15
c	20	5	3 (high)	5

- Its combined utilization is $U_C = 1.0 > U(3) = 0.78$
- The utilization-based test fails, but RTA shows that the task set will meet all its deadlines

Sporadic tasks and other extensions

- Sporadic tasks have a ***minimum inter-arrival time***
  - This should be preserved at run time if schedulability is to be ensured, but how can it ?
- The RTA for FPS works perfectly well for $D \le T$ as long as the stopping criterion becomes $W_i^{n+1} > D_i$
- Interestingly, RTA also works perfectly well with *any* priority ordering, as long as the task indices reflect it

Coexistence of hard and soft tasks /1

- In many situations, the WCET given for sporadic tasks are considerably higher than the average case
  - The WCET values are far off the center of the Gaussian
- In exceptional circumstances, interrupts may arrive in bursts, and abnormal sensor readings may require significant extra computation to restore a baseline truth
- Analyzing feasibility with WCET may lead to very low processor utilization at run time, *subtracted* to soft tasks
  - Hence to undesirable waste of precious (and scarce) resource and a reduction of functional throughout
- We need some common-sense rules to contain such pessimism

Coexistence of hard and soft tasks /2

- **Rule 1 :** All tasks (hard and soft; periodic and sporadic) should be schedulable using *average* execution times and *average* (sporadic) arrival rates
  - Hence, there may be situations where it may *not* be possible to meet all deadlines
  - This condition is known as a *transient overload*
    - It is transient so long as not all tasks transition forever to worst-case behavior
- **Rule 2 :** All hard real-time tasks should be schedulable using WCET and worst-case arrival rates of all tasks (including soft)
  - No hard real-time task will therefore miss its deadline
  - If Rule 2 causes unacceptably low utilization for soft tasks then WCET values or arrival rates should be "massaged"

Handing aperiodic tasks /1

- They do *not* have minimum inter-arrival times, and consequently *cannot* claim deadlines
  - We may be interested in the system being responsive to them
  - In cyclic scheduling we would use *slack stealing* for those tasks ….
- We might run aperiodic tasks at a priority below the priorities assigned to hard tasks
  - That way, under preemption, aperiodic tasks won't be able to steal resources from hard tasks
- But this solution would penalize soft tasks, which might miss their deadlines too often
- We need another kind of solution …

Handing aperiodic tasks /2

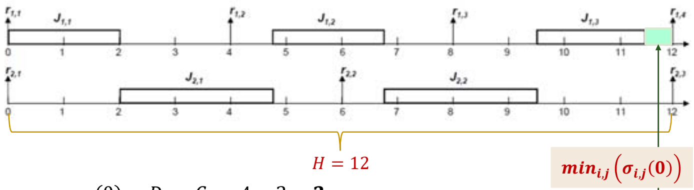
- … A solution that, besides preserving hard tasks and giving fair opportunities to soft tasks, should minimize
  - The response time of the job *at the head* of the aperiodic queue
  - Or the average response time of *as many* aperiodic jobs as possible for a given queuing discipline
- Possible choices
  - Execute the aperiodic jobs in the background
  - Execute the aperiodic jobs by interrupting the periodic jobs
  - Use slack stealing
  - Use dedicated servers

Handing aperiodic tasks /3

- ***Slack stealing***
  - Difficult to implement for preemptive systems
    - The slack $\sigma(t)$ is a *not* a constant for them
    - It is a function of the time t at which it is computed
  - The slack stealer is ready when the aperiodic queue is not empty; it is suspended otherwise
  - When ready and $\sigma(t) > 0$, the slack stealer is assigned the highest priority; the lowest when $\sigma(t) = 0$
  - Static computation of $\sigma(t)$ for some t is useful but only when the release jitter in the system is very low
    - Under EDF, $\sigma(t = 0) = min_i\{\sigma_i(0)\}$ where $\sigma_i(0) = D_i - \sum_{k=1,..,i} e_k$ for *all* jobs released in the hyperperiod starting at $t = 0$

Computing the slack under EDF

$T_1 = (4, 2)$, $T_2 = (6, 2.75)$ - EDF scheduling: $(\cancel{\phi_i}, p_i, e_i, \cancel{D_i})$



$\sigma_{1,1}(0) = D_1 - C_1 = 4 - 2 = \mathbf{2}$
$\sigma_{2,1}(0) = D_2 - C_1 - C_2 = 6 - 2 - 2.75 = \mathbf{1.25}$
$\sigma_{1,2}(0) = D_{1_2} - 2 \times C_1 - C_2 = 8 - 2 \times 2 - 2.75 = \mathbf{1.25}$
$\sigma_{2,2}(0) = D_{2_2} - 2 \times C_1 - 2 \times C_2 = 12 - 2 \times 2 - 2 \times 2.75 = \mathbf{2.5}$
$\sigma_{1,3}(0) = D_{1_3} - 3 \times C_1 - 2 \times C_2 = 12 - 3 \times 2 - 2 \times 2.75 = \boxed{\mathbf{0.5}}$

Computing the slack under FPS /1

- The amount of slack that an FPS system has in a time interval may depend on *when* the slack is used
- To minimise the response time of an aperiodic job J_a, the decision of when to schedule it, must consider the execution time of J_a
  - *No* slack stealing algorithm under FPS can minimise the response time of *every* aperiodic job, even with prior knowledge of their arrival and execution times
  - Better *not* be greedy in using the available slack

Computing the slack under FPS /2

- The slack of periodic jobs of τ_i should be computed based on their *effective deadline* D_i^e
  - For a job of τ_i, it should be computed at the beginning of the level-$i-1$ busy period that precedes D_i so that $D_i^e \leq D_i$
- The initial slack $\sigma_{i,j}(0)$ of every periodic job J_{ij} (the j^{th} job of task J_i) in H is determined as

$$max\left(0, D_{ij}^e - \sum_{k=1}^{i} \left\lfloor \frac{D_{ij}^e}{T_k} \right\rfloor C_k\right)$$

Slack stealing defeats optimality

- *Greed is no good* for aperiodic tasks
  - To minimize the response time of an aperiodic job, it may be necessary to schedule it later, even if slack is currently available
  - For any periodic task set, under FPS, and any aperiodic queuing policy, *no* valid algorithm exists that minimizes the response time of *all* aperiodic jobs
  - Similarly, no valid algorithm exists that minimizes the average response time of the aperiodic jobs

T.-S. Tia, J. W.-S. Liu, and M. Shankar, "Algorithms and Optimality of Scheduling Aperiodic Requests in Fixed-Priority Preemptive Systems," Journal of Real-Time Systems, 10(1), pp. 23-43, 1996.

Handing aperiodic tasks /4

- ***Periodic server*** (PS), a general model
  - The PS is a notional (T_{ps}, C_{ps}) periodic task scheduled at the highest priority solely to execute aperiodic jobs
    - The PS has a ***budget*** C_{ps} time units and a ***replenishment period*** of length T_{ps}
    - When the PS is scheduled and executes aperiodic jobs, it consumes its budget at the rate of 1 unit per unit of time
    - Budget is exhausted when $C_{ps} = 0$ and replenished periodically
  - The PS is *backlogged* when the aperiodic job queue is nonempty and it is idle otherwise
    - Eligible for execution only when ready, backlogged and $C_{ps} > 0$

Handing aperiodic tasks /5

- ***Polling server***, a simple (naïve) kind of PS
  - It is given a fixed budget that it uses to serve aperiodic task requests that is replenished at every period
  - The budget is immediately consumed if the server is scheduled while idle
  - It is *not* ***bandwidth preserving***, hence it is inefficient
    - An aperiodic job that arrives just after the server has been scheduled while idle, must wait until the next replenishment time
  - Bandwidth-preserving servers need additional rules for consumption and replenishment of their budget

Handing aperiodic tasks /6

- ***Deferrable Server*** (DS), a *bandwidth-preserving* PS
  - DS retains its budget if no aperiodic tasks require execution
    - If an aperiodic job requires execution during the DS period, it can be served immediately: when idle, the DS stays ready
  - The budget is replenished at the start of the new period (!)
    - If an aperiodic job arrives ε time units before the end of T_{ds}, the request begins to be served and blocks periodic tasks
    - When the budget is replenished, new aperiodic jobs may then be served for the full budget
  - If that happens, in $\omega(t)$, DS contributes a solid interference of $C_{ds} + \left\lceil \frac{t - C_{ds}}{T_{ds}} \right\rceil C_{ds}$, *longer* than $1 \times C_{ds}$ per busy period

Handing aperiodic tasks /7

- ***Sporadic Server*** (SS), fixes the bug in DS
  - The budget is replenished <u>only when exhausted</u> and at a minimum guaranteed distance from its earlier execution
    - Hence no longer at a fixed rate
  - This places a tighter bound on its interference and makes schedulability analysis simpler and less pessimistic
- This is the default server policy in POSIX

SS rules under FPS

- ***Consumption rules***
  - At time $t > t_r$ (the latest replenishment time), a backlogged SS consumes budget only if executing, hence when no higher-priority task is ready
  - The replenishment is limited to the quantity of actual consumption
- ***Replenishment rules***
  - t_r records the time that SS' budget was last replenished
  - t_e records the time when SS first begins to execute since t_r
    - $t_e > t_r$ is the latest time at which a lower-priority task than SS executes
  - The next replenishment time is set to $t_e + T_{ss}$
- ***Exception***
  - If only higher-priority tasks had been busy since t_r, then $t_e + T_{ss} > t_r + T_{SS}$ and SS is late: hence, budget fully replenished as soon as exhausted

SS rules unveiled

- Let t_a be the time at which SS has full budget *and* becomes backlogged, and $t_f \geq t_a$ the time at which SS becomes idle
- In the $[t_a, t_f]$ interval, when SS is continuously active, three cases are possible

1. SS has consumed no capacity: $t_{r_{next}} = t_f + T_{SS}$ → no replenishment, and no interference in that interval
2. SS has consumed all capacity: $t_{r_{next}} = t_a + T_{SS}$ → full replenishment, and bounded interference in that interval
3. SS has consumed fractional capacity: $t_{r_{next}} = t_f + T_{SS}$ → fractional replenishment, and interference lower than allowed in that interval

Handing aperiodic tasks /8

- SS is more complex than PS or DS
  - Its rules require keeping tab of lots of data
  - Several cases to consider when making scheduling decisions
  - This complexity is acceptable because the schedulability of a SS is easy to demonstrate
    - Under FPS, SS equates to a periodic task τ_s with (p_s, e_s)
- EDF and LLF use a dynamic variant of SS as well as other bandwidth-preserving server algorithms known as
  - *Constant utilization server*
  - *Total bandwidth server*
  - *Weighted fair queuing server*

Task sets with D < T

- We know that, for $D = T$, Rate Monotonic priority assignment (aka RMS) is optimal
- For $D < T$, ***Deadline Monotonic*** priority ordering (DMPO), where $D_i < D_j \rightarrow P_i > P_j$, is optimal
  - Any task set Q that is schedulable by priority-driven scheme W, it is also schedulable by DMPO
- The proof of optimality of DMPO involves transforming the priorities of Q as assigned by W until the ordering becomes as assigned by DMPO
  - Each step of the transformation preserves schedulability

DMPO is optimal /1

- Let τ_i, τ_j be two tasks with adjacent priorities in Q such that under W we have $P_i > P_j \ \wedge \ D_i > D_j$
- Define scheme W' to be identical to W except that tasks τ_i, τ_j are swapped
- Now consider the schedulability of Q under W'
- All tasks $\{\tau_k\}$ with priority $P_k > P_j$ will be unaffected
- All tasks $\{\tau_s\}$ with priority $P_s < P_i$ will be unaffected as they will experience the same interference from τ_j and τ_i
- Task τ_j which was schedulable under W, now has a higher priority, suffers less interference, and hence must be schedulable under W'

DMPO is optimal /2

- All that is left to show is that task τ_i, which has had its priority lowered, is still schedulable
- Under W we have $R_j \leq D_j, D_j < D_i$ and $R_i \leq T_i$
- Task τ_j only interferes once during the execution of task τ_i hence $R_i{}' = R_j \leq D_j < D_i$
  - Under W' task τ_i completes at the time task τ_j did under W
  - Hence task τ_i is still schedulable after the switch
- Priority scheme W' can now be transformed to W'' by choosing two more tasks that are in the wrong order for DMPO and switching them

Generalized priority assignment (aka simulated annealing)

Theorem: If task p is assigned the lowest priority and it is feasible, then, if a feasible priority ordering exists for the complete task set, one such ordering exists where task p is assigned the lowest priority

```
procedure Assign_Pri (Set : in out Task_Set;
                      N   : Natural; -- number of tasks
                      OK  : out Boolean) is
begin
  for K in 1..N loop
    for Next in K..N loop
      Swap(Set, K, Next);
      Process_Test(Set, K, OK); -- is task K feasible now?
      exit when OK;
    end loop;
    exit when not OK; -- failed to find a schedulable task
  end loop;
end Assign_Pri;
```


Summary

- A simple (periodic) workload model
- Delving into fixed-priority scheduling
- A (rapid) survey of schedulability tests for FPS
- Some extensions to the workload model
- Priority assignment techniques

Selected readings

- N.C. Audsley, A. Burns, R.I. Davis, K.W. Tindell, A.J. Wellings (**1995**)
  Fixed priority pre-emptive scheduling: an historical perspective
  DOI: 10.1007/BF01094342
- D. Faggioli, M. Bertogna, F. Checconi (**2010**)
  Sporadic Server revisited
  DOI: 10.1145/1774088.1774160