CONVERGENCE TO EQUILIBRIA IN PLURALITY VOTING

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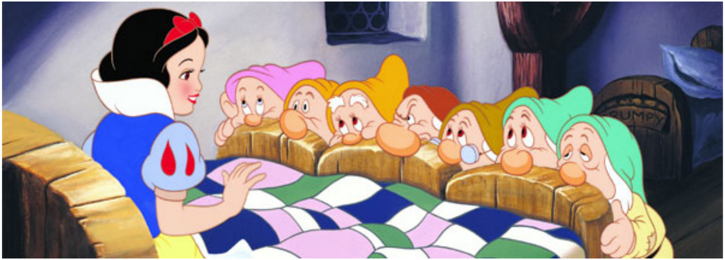
Once upon a time...

"Life is worthless without love!"
- told Snow White the Seven Dwarfs



Once upon a time...

...and fell asleep

Once upon a time...

...for a long-long while.

Who will wake Snow White up?

- Prince (P)

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- Prince (P)
- Prince Charming (C)


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- Little Prince (L)

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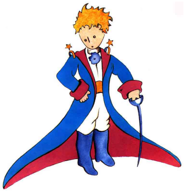
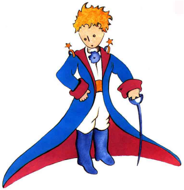


Who will wake Snow White up?

- Prince (P)
- Prince Charming (C)
- Little Prince (L)
- Batman (B)






The dwarfs have to choose

So they vote:

P		C		L		B
1	2	3	4	5	6	7

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P, C, L are tied (2 points)

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C is elected (ties broken alphabetically)

The dwarfs have to choose

So they vote:

P		C		L		B
1	2	3	4	5	6	7

C is elected (ties broken lexicographically)

But wait a minute...

The voters have the following preferences regarding the outcome:

$$
\begin{array}{ll}
1: & P \succ B \succ L \succ C \\
2: & P \succ B \succ C \succ L \\
3: & C \succ L \succ P \succ B \\
4: & C \succ B \succ P \succ L \\
5: & L \succ \ldots \\
6: & L \succ \ldots \\
7: & B \succ \ldots
\end{array}
$$

The dwarfs have incentives to strategise

So they may change their mind:

P	C		L		B	
2	3	4	5	6	1	7

C, L, B are tied (2 points)

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So they may change their mind:

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B is elected (lexicographic tie-breaking)

… and change the outcome!

It's not yet the end...

The voters have the following preferences regarding the outcome:

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$$

The dwarfs have incentives to strategise

So they may change their mind:

P	C	L			B	
2	4	3	5	6	1	7

L is elected (unique winner)

It's not yet the end...

The voters have the following preferences regarding the outcome:

$$
\begin{aligned}
1 &: \quad P \succ B \succ L \succ C \\
2 &: \quad P \succ B \succ C \succ L \\
3 &: \quad C \succ L \succ P \succ B \\
4 &: \quad C \succ B \succ P \succ L \\
5 &: \quad L \succ \ldots \\
6 &: \quad L \succ \ldots \\
7 &: \quad B \succ \ldots
\end{aligned}
$$

The dwarfs have incentives to strategise

So they may change their mind:

P	C	L			B		
	4	3	5	6	1	2	7

B is elected (lexicographic tie-breaking)

No more objections!

P	C	L			B		
	4	3	5	6	1	2	7

$$
\begin{aligned}
1 &: \quad P \succ B \succ L \succ C \\
2 &: \quad P \succ B \succ C \succ L \\
3 &: \quad C \succ L \succ P \succ B \\
4 &: \quad C \succ B \succ P \succ L \\
5 &: \quad L \succ \dots \\
6 &: \quad L \succ \dots \\
7 &: \quad B \succ \dots
\end{aligned}
$$

Happy end!

What are we after?

- Agents have to agree on a joint plan of action or allocation of resources.
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Voting setting

- $V = \{1, \ldots, n\}$ – set of *voters* (or *agents*)
- $C = \{c_1, \ldots, c_m\}$ – set of *candidates* (or *alternatives*)
- $\mathcal{L}(C)$ – set of all *strict linear orders* (transitive, antisymmetric and total relations) on C
- $\succ_i \in \mathcal{L}(C)$ – agent i's private *preference order* over the candidates, for each $i \in V$

Voting profile

- $P_i \in \mathcal{L}(C)$ – *vote* of voter i (may or may not coincide with $\succ_i$)
- $P = (P_1, \ldots, P_n) \in \mathcal{L}(C)^n$ – voting *profile*
    - $P = (P_i, P_{-i})$ where P_{-i} – set of partial votes of a subset $V \setminus \{i\}$ of all the agents but i
- $P = (\succ_1, \ldots, \succ_n)$ – *truthful* profile

Voting rule

- $F : \mathcal{L}(C)^n \to 2^C \setminus \{\emptyset\}$ – *voting rule*
  - determines *the winners* of the election

Voting rule

- $F : \mathcal{L}(C)^n \to C$ – *resolute* voting rule
  - returns a single winner
  - paired with a *tie-breaking rule*
    - deterministic (e.g., lexicographic)
    - randomised

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Plurality

- Each voter reports his top candidate:
  - $P_i \in C$
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$$s(c) = \sum_{i \in V : P_i = c} w_i$$

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Game-theoretical interpretation

The (Plurality) voting game is a normal form game $\langle V, C, F, \succ \rangle$ where:

- V – set of agents $=$ set of voters
- C – set of strategies $=$ set of candidates
- F – voting rule (paired with a tie-breaking rule)
- $\succ$ – profile of voters' preferences over the candidates

Game-theoretical interpretation

The (Plurality) voting game is a normal form game $\langle K, C, F, \succ\rangle$ where:

- $K \subseteq V$ – set of agents = set of strategic voters
- $B = V \setminus K$ – sincere (non-strategic) voters
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Voting as a normal form game

3 candidates with initial scores:

- $s_B(a) = 7$
- $s_B(b) = 9$
- $s_B(c) = 3$

$w_1 = 3$ \ $w_2 = 4$	a	b	c
a			
b			
c			

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b	(11, 12, 3)		
c			

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a	(14, 9. 3)	(10, 13, 3)	(10, 9. 7)
b	(11, 12, 3)	(7, 16, 3)	(7, 12, 7)
c	(11, 9. 6)	(7, 13, 6)	(7, 9, 10)

Voting as a normal form game

Agents' preferences:

- $1 : a \succ b \succ c$
- $2 : c \succ a \succ b$

$w_1 = 3$ \ $w_2 = 4$	a	b	c
a	(14, 9. 3)	(10, 13, 3)	(10, 9. 7)
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Voting as a normal form game

Agents' preferences:

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$w_2 = 4$ / $w_1 = 3$	a	b	c
a	**(14, 9. 3)**	(10, 13, 3)	**(10, 9. 7)**
b	(11, 12, 3)	**(7, 16, 3)**	(7, 12, 7)
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Voting in turns (a.k.a. "iterative voting")

- Agents start from some initial profile (e.g., truthful).
- They change their votes in turns.
- At each step, a single agent makes a move.
- The game ends when there are no more objections.

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Improvement moves

Agents make rational moves to improve their state, when

- they do not know the preferences of the others,
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$$3: \quad C \succ L \succ P \succ B$$

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$B \xrightarrow{3} L$ is an *improvement move* (or *better reply*) of agent 3

Improvement moves

$3: \quad C \succ L \succ P \succ B$

P		C		L		B
1	3	2	4	5	6	7

$B \xrightarrow{3} P$ is a *best reply* of agent 3

Improvement moves

$$3 : \quad C \succ L \succ P \succ B$$

P	C			L		B
1	2	3	4	5	6	7

$B \xrightarrow{3} C$ is a *restricted best reply* (which is unique) for agent 3

Variations of the game

Voting setting:

- Voting rule
  - Plurality

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- Tie-breaking rule
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- Agent types
  - Weighted
  - Unweighted

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Dynamics:

- Initial state
  - Truthful
  - Arbitrary

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  - Better replies
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Our results

We show how the convergence depends on *all* of these game/dynamic attributes.

Deterministic tie-breaking

Theorem

If all agents have weight 1 and use restricted best replies, the game converges to a Nash equilibrium from any state.

Proof sketch

(by Reyhani & Wilson 2012)

- o_t – outcome at step t
- Restricted best replies at any step t are of 2 types:
  - **type 1:** $a \to b$ where $a \neq o_{t-1}$ and $b = o_t$
  - **type 2:** $a \to b$ where $a = o_{t-1}$ and $b = o_t$
- We will show that there are
  - $\leq m$ moves of type 1 in total, and
  - $\leq m - 1$ moves of type 2 for each voter.

Proof

$PW_t = \{c \big| \exists i \in K : o_t \xrightarrow{i} c \Rightarrow c = o_{t+1}\}$ – potential winners at step t

Lemma

For $t < t'$ we have $PW_{t'} \subseteq PW_t$.

Proof of the lemma

- Let $a \to b$ at step t. Then, $b = o_t$.
- Let $c \in PW_t$.
- Consider the scores of $b, c, y \; \forall y \in C \setminus \{a, b\}$:

$$s_{t-1}(c) + 1 = s_t(c) + 1 \succeq s_t(b) - 1 = s_{t-1}(b)$$
$$s_{t-1}(c) + 1 = s_t(c) + 1 \succeq s_t(y) = s_{t-1}(y)$$

where $c \succeq c'$ if $s(c) > s(c')$ or $s(c) = s(c')$ and c has a lower index.

Proof of the lemma (contd.)

- If $a \to b$ at step t is of type 2, then followed by $b \to c$ at step $t+1$ results in the same scores as $a \to c$ at step t. Hence, $c \in PW_{t-1}$.
- Otherwise, let $a' = o_t$ and note $a' \neq a, b$.
- We have:

$$\begin{aligned} s_{t-1}(c) + 1 &\succeq s_{t-1}(a') \\ s_{t-1}(a') &\succeq s_{t-1}(y) \qquad \forall y \in C \\ \Rightarrow \quad s_{t-1}(c) + 1 &\succeq s_{t-1}(a') \succeq s_{t-1}(a) \end{aligned}$$

Hence, $c \in PW_{t-1}$.

□

Proof of the theorem (contd.)

- If $a \rightarrow b$ at step t is of type 1 then $a \notin PW_t$:
  - If $a \in PW_t$ then $b \rightarrow a$ makes a a winner, a contradiction to $a \rightarrow b$ being of type 1.
- By the lemma, $a \notin PW_{t'}$ for all $t' > t$
  $\Rightarrow$ the number of type 1 moves is bounded by m.

- At every improvement step $a \xrightarrow{i} b$ of type 2, it must hold that $b \succ_i a$
  $\Rightarrow$ each voter can make at most $m - 1$ steps of type 2.

$\square$

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$\square$

(Not restricted) best replies

3 candidates with initial scores $(1, 0, 0)$

2 voters with preferences

- $1 : a \succ b \succ c$
- $2 : c \succ b \succ a$

voter 1 \ voter 2	a	b	c
a			
b		(1,2,0)	(1,1,1)
c		(1,1,1)	(1,1,1)

Cycle from an arbitrary state!

Better replies

4 candidates with initial scores $(2, 2, 2, 0)$

3 voters with preferences

- $1, 3 : d \succ a \succ b \succ c$
- $2 : c \succ b \succ a \succ d$

$$dcd(2, 2, 3, 2) \xrightarrow{1} bcd(2, 3, 3, 1) \xrightarrow{3} bca(3, 3, 3, 0)$$

Better replies

4 candidates with initial scores $(2, 2, 2, 0)$

3 voters with preferences

- $1, 3 : d \succ a \succ b \succ c$
- $2 : c \succ b \succ a \succ d$

$$dcd(2, 2, 3, 2) \xrightarrow{1} bcd(2, 3, 3, 1) \xrightarrow{3} bca(3, 3, 3, 0)$$

$$\uparrow_1$$

$$\xrightarrow{2} bba(3, 4, 2, 0) \xrightarrow{1} cba(3, 3, 3, 0) \xrightarrow{2} cca(3, 2, 4, 0)$$

Cycle from the truthful state!

Better replies

4 candidates with initial scores $(2,2,2,0)$

3 voters with preferences

- $1,3 : d \succ a \succ b \succ c$
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$$\uparrow_1$$

$$\xrightarrow{2} bba(3,4,2,0) \xrightarrow{1} cba(3,3,3,0) \xrightarrow{2} cca(3,2,4,0)$$

Cycle from the truthful state!

Weighted voters

- No convergence for 3+ voters, even when
  start from the truthful state and use restricted best replies
- Convergence for 2 voters, if they both
  use restricted best replies **or** start from the truthful state

Randomised tie-breaking

- $\succ_i$ does not induce a complete order over the oucomes, which are *sets* of candidates.
- We augment agents' preferences with cardinal utilities:
  - $u_i(c) \in \mathbb{R}$ – utility of candidate c to voter i,
  - for multiple winners, $u_i(W) = \frac{\sum_{c \in W} u_i(c)}{|W|}$.
- A utility function u is *consistent* with a preference relation $\succ_i$ if

$$u(c) > u(c') \Leftrightarrow c \succ_i c'$$

Randomised tie-breaking

- To prove convergence, we must show it is guaranteed for *any* utility function which is consistent with the given preference order.
- To disprove, it is sufficient to show a cycle for a *specific* assignment of utilities: *weak* counterexample.
- If the counterexample holds for any profile of utility scales, it is *strong*.

Weighted voters

3 candidates with initial scores $(0, 1, 3)$

2 voters weighted voters with preferences

- $1 : a \succ b \succ c$
- $2 : b \succ c \succ a$

$w_1 = 5$ \ $w_2 = 3$	a	b	c
a	(8,1,3)	(5,4,3)	(5,1,6)
b	(3,6,3)	(0,9,3)	(0,6,6)
c	(3,1,8)	(0,4,8)	(0,1,11)

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No Nash equilibrium!

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c	(3,1,8)	(0,4,8)	(0,1,11)

No Nash equilibrium!

Unweighted voters

Theorem

If all agents have weight 1 and use restricted best replies,
the game converges to a Nash equilibrium from the truthful state.

Proof (skipped)

We show that in each step, an agent votes for a less preferred candidate.

- Clearly holds for the first step. Proceed by induciton.

Hence, each voter can make only $m - 1$ steps.

Less restricted dynamics

- Arbitrary state:
  - weak counterexample with 3 unweighted agents, even if they use restricted best replies
- Better replies:
  - strong counterexample with 3 unweighted agents
  - weak counterexample with 2 agents, even if they start from the truthful state

Summary

Deterministic Tie breaking

Dynamics / Initial state	R. best reply: Truth	R. best reply: Any	Best reply: Truth	Best reply: Any	Any better reply: Truth	Any better reply: Any
Weighted $(k > 2)$	X	X	X	X	X	X
Weighted $(k = 2)$	V	V	V	X	V	X
Non-weighted	V	V		X	X	X

Randomized Tie breaking

Dynamics / Initial state	R. best reply: Truth	R. best reply: Any	Any better reply: Truth	Any better reply: Any
Weighted	X	X	X	X
Non-weighted	V	X	X	X

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Deterministic Tie breaking

Dynamics / Initial state	R. best reply: Truth	R. best reply: Any	Best reply: Truth	Best reply: Any	Any better reply: Truth	Any better reply: Any
Weighted $(k > 2)$	X	X	X	X	X	X
Weighted $(k = 2)$	V	V	V	X	V	X
Non-weighted	V	V	V (#)	X	X	X

(#) Reijngoud & Endriss, 2012

Randomized Tie breaking

Dynamics / Initial state	R. best reply: Truth	R. best reply: Any	Any better reply: Truth	Any better reply: Any
Weighted	X	X	X	X
Non-weighted	V	X	X	X

Other remarks

- Truth biased agents
  - Vangelis's talk tomorrow

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    - Vangelis's talk tomorrow
- Quality of outcomes

Other remarks

- Truth biased agents
  - Vangelis's talk tomorrow
- Quality of outcomes
  - Simina's talk tomorrow

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Future work

Past future work

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- Rules other than Plurality
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- Weak acyclicity?
- Dynamics leading to desirable outcomes?

THE END