Pre-vote negotiations and the outcome of collective decisions

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joint work with

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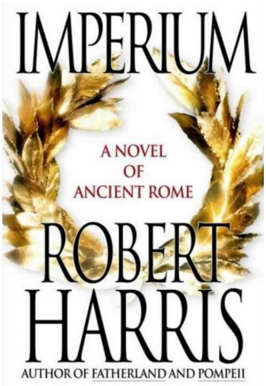
Bits of Roman politics

Cicero used to say that it was not in the senate chamber that the real business of the republic was done, but outside, in the open-air lobby known as the senaculum, where the senators were obliged to wait until they constituted a quorum.



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- Capture the structure of negotiations before a collective decision (lobbying, do ut des);
- Understand how pre-vote negotiations affect collective decisions (achievable (un)desirable properties);
- Ideally, a framework for political analysis.

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- *Voting* games with resources and pre-vote negotiations;
  - possibility of investing resources beforehands to convince the others to change their vote;
  - goals as ideological positions (that cannot be changed by monetary offers).

Voting games and goals

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Voting games and goals

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- issues are logically interdependent, and might be subjected to satisfy a given formula, i.e., a given *integrity constraint*.

A key reference

Umberto Grandi and Ulle Endriss
Lifting integrity constraints in binary aggregation.
Artificial Intelligence 199-200:45–66. 2013.

Voting

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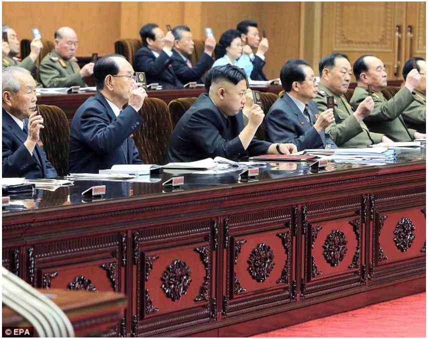
- $\mathcal{N} = \{1, \ldots, n\}$ is a finite set of individuals;
- $\mathcal{I} = \{1, \ldots, m\}$ is a finite set of issues;
- IC is a propositional formula of $\mathcal{L}_{PS}$, a propositional language constructed over the set $PS = \{p_1, \ldots, p_m\}$.

Atomic weapons

Example

A parliament is to decide whether to build nuclear power plants (N) and develop atomic weapons (W). If importing nuclear technology from abroad is not an option, the development of atomic weapons involves the construction of nuclear power plants, i.e., IC $= (W \rightarrow N)$.



Definition (Aggregation procedure)

An *aggregation procedure* for BA structure $\mathcal{S}$ is a function

$$F : \mathrm{Mod}(\mathsf{IC})^{\mathcal{N}} \to \mathcal{D}$$

mapping every profile of IC-consistent ballots $(\mathrm{Mod}(\mathsf{IC})^{\mathcal{N}})$ to a binary ballot $(\mathcal{D})$.

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- Majority, unanimity etc.
- APs can be studied axiomatically.

Question

- If individuals provide an IC-consistent ballot, will the resulting ballot be IC-consistent as well?

Discursive Dilemma

Example

If buying nuclear energy from the foreign market is an option, i.e., it is possible to vote on the issue $(W \rightarrow N)$, there is a *natural* IC in $(W \wedge (W \rightarrow N)) \rightarrow N$, when ballot $(1, 1, 0)$ is outright inadmissible. In a parliament of 3 equally representative parties a Discursive Dilemma can arise with majority voting.

$$\text{IC} = (W \wedge (W \rightarrow N)) \rightarrow N$$

	W	$W \rightarrow N$	N
Party A	1	1	1
Party B	1	0	0
Party C	0	1	0
Majority	**1**	**1**	**0**

Table: A Discursive Dilemma

Voting Games and Goals

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Voting Games and Goals

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- Players' goals are on the outcome of the vote.

Key reference

Paul Harrenstein, Wiebe van der Hoek, John-Jules Meyer and Cees Witteveen
Boolean games.
TARK 2001

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An aggregation game is a tuple $\mathcal{A} = \langle \mathcal{N}, \mathcal{I}, \mathrm{IC}, F, \{\gamma_i\}_{i \in \mathcal{N}} \rangle$ such that:

All individuals share the same set of IC-consistent strategies, namely the set of IC-consistent ballots $\mathrm{Mod}(\mathrm{IC})$.

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- F is an aggregation procedure for $\langle \mathcal{N}, \mathcal{I}, \mathsf{IC} \rangle$;
- each γ_i is a propositional formula in $\mathcal{L}_{PS}$ which is consistent with IC;

All individuals share the same set of IC-consistent strategies, namely the set of IC-consistent ballots $\mathrm{Mod}(\mathsf{IC})$.

Preferences

Definition (Preferences in aggregation games)

Let $\mathcal{A} = \langle \mathcal{N}, \mathcal{I}, \mathrm{IC}, F, \{\gamma_i\}_{i \in \mathcal{N}} \rangle$ be an aggregation game, For ballots B, B'

$$B \succeq_i^{\mathcal{A}} B' \Leftrightarrow B' \models \neg\gamma_i \text{ or } B \models \gamma_i$$

Voting strategies

Definition

A strategy $B \in \mathrm{Mod}(\mathsf{IC})$ is *truthful for agent i* if it satisfies γ_i.
A strategy profile $\boldsymbol{B} = (B_1, \dots, B_n)$ is:

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- goal-efficient if $F(\boldsymbol{B}) \models \bigwedge_i \gamma_i$;

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- goal-efficient if $F(\boldsymbol{B}) \models \bigwedge_i \gamma_i$;
- goal-inefficient if $F(\boldsymbol{B}) \not\models \gamma_i$ for all $i \in \mathcal{N}$.

Games and goals

Definition

An aggregation game is *consistent* if the conjunction of all individual goals is consistent with IC, i.e., if $(\bigwedge_{i \in \mathcal{N}} \gamma_i) \wedge \mathrm{IC}$ is consistent.

Equilibria

Proposition

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Every consistent aggregation game for the majority rule (maj) has an IC*-consistent NE that is truthful and goal-efficient.*

- Idea: at unanimous, truthful, IC-consistent and goal-efficient profile $\boldsymbol{B}^* = (B^*)^{\mathcal{N}}$, $maj(\boldsymbol{B}^*) = B^*$.
- Generalizable! (as many results next)

Equilibria

Proposition

There exist a consistent aggregation game for maj with a truthful NE that is goal-inefficient and IC*-inconsistent.*

	p_1	p_2	p_3
Voter 1	1	0	0
Voter 2	0	1	0
Voter 3	0	0	1
Majority	**0**	**0**	**0**

Table: An equilibrium with $\mathrm{IC} = p_1 \vee p_2 \vee p_3$ and $\gamma_i = p_i$

Voting games, goals and payoff

- Payoff associated to each possible vote;

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- Payoff associated to each possible vote;
- Goals and payoffs play a role in ballot selection.

Key reference

Mike Wooldridge, Ulle Endriss, Sarit Kraus and Jérôme Lang.
Incentive engineering in boolean games.
Artificial Intelligence 2013.

Aggregation games with payoff

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- A lexicographic (*quasi-dichotomous*) preference relation.

Goals, payoffs and induced preferences

Definition (Preferences in A^π games)

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- First we look at the goal, then at the payoff.

Uniform games

Definition

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- Payoff received only depends on the outcome of the vote.

Uniform payoff: properties

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- Same idea: at unanimous, truthful, IC-consistent and goal-efficient profile $\boldsymbol{B}^* = (B^*)^{\mathcal{N}}$, $maj(\boldsymbol{B}^*) = B^*$. It is an equilibrium thanks to *maj* and uniformity.

Non-uniform payoff

Proposition

For every uniform A^{π}-game $\langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ and profile $\boldsymbol{B}^*$ such that $\boldsymbol{B}^*$ is a goal-inefficient NE for $\mathcal{A}$, there exists a payoff function $\{\pi'_i\}_{i \in \mathcal{N}}$ such that:

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- $\sum_{i \in N} \pi'_i(\boldsymbol{B}) = \sum_{i \in N} \pi_i(\boldsymbol{B})$, *for every profile* $\boldsymbol{B}$;
- $\boldsymbol{B}^*$ *is not a NE for* $\langle \mathcal{A}, \{\pi'_i\}_{i \in \mathcal{N}} \rangle$.

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- Idea: each voter could *pay* the others to make their deviation to his goal state profitable for them. No matter how expensive it is, he is still going to be better off.

Pre-vote negotiations

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- A *vote phase*, where players play the original A^π-game, updated with transfers.

Key references

Paolo Turrini
Endogenous boolean games.
IJCAI 2013.

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Paolo Turrini
Endogenous boolean games.
IJCAI 2013.

Matthew O. Jackson and Simon Wilkie
Endogenous games and mechanisms: Side payments among players.
Review of Economic Studies 72(2):543–566, 2005

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- $\tau_i(\boldsymbol{B}, j)$ the amount of payoff that a player i gives to player j should a certain profile of votes $\boldsymbol{B}$ be played
- $\tau(A^\pi) = \langle \mathcal{A}, \{\pi'_i\}_{i \in \mathcal{N}} \rangle$ is the new game where π'_i is updated with the payments.

Equilibria in $\mathcal{A}^T$ games

Definition

Given a $\mathcal{A}^T$-game $\langle \mathcal{A}, \{\pi_i\}_{i\in\mathcal{N}}, \{T_i\}_{i\in N}\rangle$ we call a Nash equilibrium $\boldsymbol{B}$ of $\langle \mathcal{A}, \{\pi_i\}_{i\in\mathcal{N}}\rangle$ a *surviving Nash equilibrium* if there exist a transfer function τ and a subgame perfect equilibrium of the two-phase game such that $(\tau, \boldsymbol{B})$ is played on the equilibrium path.

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- SPEs constructed selecting:
  - a pure strategy Nash equilibrium after each transfer, whenever it exists;
  - a transfer profile, such that no profitable deviation exist for any player by changing his transfer;
- Assumption: *any deviation for a player to a game $\tau(\mathcal{A})$ with no pure strategy Nash equilibrium is never profitable.*

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- Surviving equilibria identify those electoral outcomes that can be rationally sustained by an appropriate pre-vote negotiation.

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- Surviving equilibria identify those electoral outcomes that can be rationally sustained by an appropriate pre-vote negotiation.
- We are interested to know whether desirable equilibria, e.g., goal-efficient, can be achieved or maintained in the two-phase game.

Equilibria in $\mathcal{A}^T$ games

Proposition

Let $\mathcal{A}^T = \langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}}, \{T_i\}_{i \in N} \rangle$ *be an endogenous aggregation game with more than two players. Every goal-efficient NE of* $\langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ *is a surviving equilibrium.*

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- Notice: (substantially) indepedendent of aggregation procedure and integrity constraint.

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Let $\mathcal{A}^T = \langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}}, \{T_i\}_{i \in N} \rangle$ *be an endogenous aggregation game for maj such that* $\wedge_{i \in \mathcal{N}} \gamma_i$ *is consistent. No goal-inefficient NE of* $\mathcal{A}$ *is a surviving equilibrium.*

Avoiding global inconsistency

- What happens if players want to achieve IC?

Avoiding global inconsistency

Definition (Augumented preferences in A^π games)

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Under the newly defined preference relations

Proposition

Let $\mathcal{A}^T = \langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}}, \{T_i\}_{i \in N} \rangle$ *be a consistent endogenous aggregation game with more than two players. Every goal-efficient NE of* $\langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ *is a surviving equilibrium if and only if it satisfies* IC.

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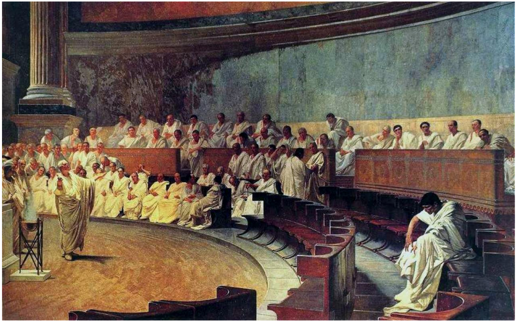

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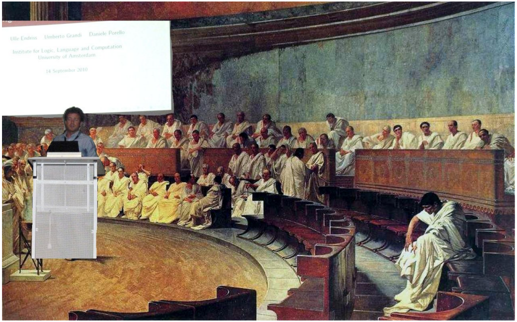

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Summarizing

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- In aggregation games equilibrium outcomes might be goal-inefficient;
- Redistributing payoff in uniform games can avoid goal-inefficient outcomes;
- Pre-vote negotiations avoid goal-inefficient outcomes, whenever goal-efficient ones are possible.

Ideas for the future

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- Different sorts of transfers (e.g., not on outcomes but on strategies of other players);
- Constraining transfers (somehow, someway);
- Milder incentives to avoid violation of integrity constraints.
- Use different IC for every voter. No logical but political dependence among issues, coherence rather than consistency!