Binary Aggregation with Integrity Constraints

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Everything Starts From Paradoxical Situations

Elections in the U.S. and in many other countries are decided using the plurality rule: the candidate who gets the most votes win.

Assume that the preferences of the individuals in Florida are as follows:

49%:	Bush $\succ$ Gore $\succ$ Nader
20%:	Gore $\succ$ Nader $\succ$ Bush
20%:	Gore $\succ$ Bush $\succ$ Nader
11%:	Nader $\succ$ Gore $\succ$ Bush

Bush results as the winner of the election, but:

- Gore wins against any other candidate in pairwise election.
- Nader supporters have an incentive to manipulate.

What About Computer Science?

Suppose three agents in a multi-agent system need to decide whether to perform a collective decision A. The decision is performed if two parameters T_1 and T_2 exceed a given threshold. Consider the following situation:

	T_1	T_2	A
Agent 1	Yes	Yes	Yes
Agent 2	No	Yes	No
Agent 3	Yes	No	No
Majority	Yes	Yes	No

Should the agents perform action A or not?

- A majority of agents think the first parameter exceeds the threshold.
- A majority of agents think the second parameter exceeds the threshold.
- But: a majority of agents think action A should not be performed!!

Outline

1. A very brief introduction to Computational Social Choice
2. Part I: Binary aggregation with integrity constraints
3. Part II: A general definition of paradox
4. Part III: Lifting rationality assumptions and axiomatic properties
5. Part IV: Can we avoid all paradoxes?
6. Directions for future research

Computational Aspects of Collective Decision Making

Different social choice problems studied:

- Choosing a winner given individual preferences over candidates.
- Allocate resources to users in an optimal way.
- Finding a stable matching of students to schools.
- Merge the results of several search engines.

Different computational techniques used:

- Algorithm design to implement complex mechanisms.
- Complexity theory to understand limitations.
- Knowledge representation techniques to compactly model problems.

Related research areas: Algorithmic Decision Theory, Algorithmic Game Theory, Computational Economics.

Y. Chevaleyre, U. Endriss, J. Lang, and N. Maudet. A Short Introduction to Computational Social Choice. Proceedings of SOFSEM-2007.

Example I: Computational Complexity as a Barrier to Manipulation

Recall the initial example: Nader supporters have an incentive to manipulate the outcome of the election and switch to vote for Gore.

How difficult is it to manipulate an election?

For certain voting rules like plurality it is easy (=polynomial). In other cases, like in Australia where a sequential rule called STV is used, this problem is computationally hard (NP-complete):

Computational complexity can thus be a barrier to manipulation.

Bartholdi, Tovey, and Trick. The Computational Difficulty of Manipulating an Election. Social Choice and Welfare, 1989.

Faliszewski and Procaccia. AI's War on Manipulation: Are We Winning? AI Magazine, 2010.

Example II: Combinatorial Voting

When the set of alternatives has a combinatorial structure, representation of preferences may be exponential! Examples:

- Electing a committee (3 members out of 8 candidates: 112 combinations)
- Multiple referendums, deciding a menu.
- Multiple variables to describe alternatives.

CS provides tools for compact representation of preferences. But how to perform collective decision making? Vote sequentially? Vote on combinations? How to elicit preferences? How hard is to compute a winner?

F. Rossi, K.B. Venable and T. Walsh. A Short Introduction to Preferences: Between Artificial Intelligence and Social Choice, 2011.

Y. Chevaleyre, U. Endriss, J. Lang, and N. Maudet. Preference Handling in Combinatorial Domains: From AI to Social Choice. AI Magazine, 29(4):37-46, 2008.

Part I:
Binary Aggregation with Integrity Constraints

A Paradoxical Example

One more look at the initial example of three agents with sensors:

	T_1	T_2	A
Agent 1	Yes	Yes	Yes
Agent 2	No	Yes	No
Agent 3	Yes	No	No
Majority	Yes	Yes	No

Two questions:

- Why is this a paradox?
- Why does this happen?

Individual Rationality in Decision Theory

The problem:	Individual choosing over a set of alternatives $\mathcal{X}$
Rational behaviour:	Maximise a weak order over $\mathcal{X}$
	(transitive, complete and reflexive binary relation)

- Linear orders to avoid ties
- Partial orders over large domains
- Acyclic relations defined from choice functions

Many Rationalities?

Judges in a court (cf. judgment aggregation):

"O.J.Simpson is guilty" "O.J.Simpson wore bloody gloves"
"If O.J.Simpson wore the glove then he is guilty"

Rational judges?
Consistent and complete judgment sets

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Rational judges?
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Committee deciding over multiple issues:

"Cut pensions" "Cut the number of MPs"
"Liberalise the market"

Rational members?
No political power to enforce all three austerity measures:
Ballots with at most 2 yes

Binary Aggregation

Ingredients:

- A finite set $\mathcal{N}$ of individuals
- A finite set $\mathcal{I} = \{1, \ldots, m\}$ of issues
- A boolean combinatorial domain: $\mathcal{D} = \{0, 1\}^{\mathcal{I}}$

Definition

An aggregation procedure is a function $F : \mathcal{D}^{\mathcal{N}} \rightarrow \mathcal{D}$ mapping each profile of ballots $\mathbf{B} = (B_1, \ldots, B_n)$ to an element of the domain $\mathcal{D}$.

Dokow and Holzman (2010), Grandi and Endriss (AAAI-2010)

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Example: Three agents with sensors

- $\mathcal{N} = \{1, 2, 3\}$
- $\mathcal{I} = \{T_1, T_2, A\}$
- Individuals submit ballots in $\mathcal{D} = \{0,1\}^3$

$B_1 = (0, 1, 0)$ the first individual think the action should not be performed.

Integrity Constraints

A propositional language $\mathcal{L}$ to express integrity constraints on $D = \{0,1\}^m$

- One propositional symbol for every issue: $PS = \{p_1, \ldots, p_m\}$
- $\mathcal{L}_{PS}$ closing under connectives $\wedge$, $\vee$,$\neg$, $\rightarrow$ the set of atoms PS

Given an integrity constraint IC $\in \mathcal{L}_{PS}$, a rational ballot is $B \in \mathrm{Mod}(\mathrm{IC})$

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Example: Three agents with sensors

Perform action A if both parameters exceed the thresholds.
Propositional constraint: $\mathrm{IC} = (p_{T_1} \wedge p_{T_2}) \rightarrow p_A$

Individual 1 submits $B_1 = (1,1,1)$: B_1 satisfies IC ✓
Individual 2 submits $B_2 = (0,1,0)$: $B_2 \models \mathrm{IC}$ ✓
Individual 3 submits $B_3 = (1,0,0)$: $B_3 \models \mathrm{IC}$ ✓

Majority aggregation outputs $(1,1,0)$: IC not satisfied.

Every individual satisfies the same rationality assumption IC...
...what about the collective outcome?

Definition

A paradox is a triple $(F, \mathbf{B}, IC)$*, where:*

- F *is an aggregation procedure*
- $\mathbf{B} = (B_1, \dots, B_n)$ *a profile*
- $IC \in \mathcal{L}_{PS}$ *an integrity constraint*

such that $B_i \models IC$ *for all* $i \in \mathcal{N}$ *but* $F(\mathbf{B}) \not\models IC$*.*

Part I:
A Common Definition of Paradox

Condorcet Paradox

Preference aggregation (Condorcet, 1785, Arrow, 1963) studies how to obtain a collective ranking of alternatives from individual preferences. Used in voting, political theory, and CS (e.g. aggregate rankings of search engines).

In 1785 Monsieur le Marquis de Condorcet pointed out that:

$$\begin{array}{c} \triangle \succ_1 \bigcirc \succ_1 \square \\ \square \succ_2 \triangle \succ_2 \bigcirc \\ \bigcirc \succ_3 \square \succ_3 \triangle \\ \hline \triangle \succ \bigcirc \succ \square \succ \triangle \end{array}$$

The collective ranking is a cycle!

Preference Aggregation

Linear order $<$ over alternatives $\mathcal{X}$ $\Leftrightarrow$ Ballot $B_{\leqslant}$ over issues $\mathcal{I} = \{ab \mid a \neq b \in \mathcal{X}\}$

$\mathrm{IC}_<$ encodes the rationality assumption for decision theory:

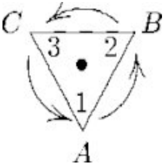
Irreflexivity: $\neg p_{aa}$ for all $a \in \mathcal{X}$

Completeness: $p_{ab} \vee p_{ba}$ for all $a \neq b \in \mathcal{X}$

Transitivity: $p_{ab} \wedge p_{bc} \rightarrow p_{ac}$ for $a, b, c \in \mathcal{X}$ pairwise distinct

Condorcet Paradox Revisited

	$\triangle\bigcirc$	$\bigcirc\square$	$\triangle\square$
Agent 1	1	1	1
Agent 2	0	1	0
Agent 3	1	0	0
Maj	1	1	0



Our definition of paradox:

- F is issue by issue majority rule
- the profile is the one described in the table
- IC that is violated is $p_{\triangle\bigcirc} \wedge p_{\bigcirc\square} \rightarrow p_{\triangle\square}$

Judgment Aggregation

Judgment Aggregation (Kornhauser and Sager, 1986, List and Pettit, 2002) studies the aggregation of judgments over a set of correlated propositions.

The discursive dilemma is a paradoxical situation that can arise when a court of judges decides on more than three correlated issues:

> *Suppose an individual is considered guilty if she has broken a contract and the contract was valid. Call α the proposition "the contract was broken", β the proposition "the contract was valid". The individual is guilty if and only if $\alpha \wedge \beta$ is judged true. Consider the following situation...*

Kornauser and Sager. Unpacking the court. Yale Law Journal, 1986.

Discursive Dilemma

	α	β	$\alpha \wedge \beta$
Agent 1	1	1	1
Agent 2	0	1	0
Agent 3	1	0	0
Majority	1	1	0!!

Our definition of paradox:

- F is issue by issue majority rule
- profile described in the table
- IC that is violated is $\neg(p_\alpha \wedge p_{\neg\beta} \wedge p_{(\alpha\wedge\beta)})$

Common feature: Three issues

The Common Structure of Paradoxes

Proposition

The majority rule does not generate a paradox with respect to IC if and only if IC is equivalent to a conjunction of clauses of size $\leqslant 2$.

$$\mathcal{IC}(\mathit{Maj}) = \text{2-CNF}$$

Common feature of all paradoxes:
clauses of size 3 are not lifted by majority

Umberto Grandi, The Common Structure of Paradoxes in Aggregation Theory, COMSOC-2012.

Part II:
Lifting Rationality Assumptions

Languages, Axioms, and Collective Rationality

Several languages for integrity constraints:

- *cubes*: conjunctions
- *k-pclauses*: positive disjunctions of size $\leqslant k$
- *XOR*: conjunctions of $p \leftrightarrow \neg q$
- ...

Definition

F is collectively rational (CR) for IC $\in \mathcal{L}_{PS}$ if for all profiles $\mathbf{B}$ such that $\mathbf{B}_i \models$ IC for all $i \in N$ then $F(\mathbf{B}) \models$ IC.

Several axioms to classify aggregation procedures:

- Unanimity (U): For any profile $\mathbf{B}$ and any $x \in \{0,1\}$, if $\mathbf{B}_{i,j} = x$ for all $i \in N$, then $F(\mathbf{B})_j = x$.
- Independence, Neutrality...

Characterisation Results: Examples

Characterisation results:

Proposition

$\mathcal{CR}[cubes] = \mathcal{F}_{cubes}[Unanimity]$. In words: F is CR with respect to all cubes iff it is unanimous

Similar results for equivalences (issue-neutrality), XOR formulas (domain-neutrality), positive implications (neutral-monotonicity).

Negative result:

Proposition

There is no language $\mathcal{L} \subseteq \mathcal{L}_{PS}$ such that $\mathcal{CR}[\mathcal{L}] = \mathcal{F}_{\mathcal{L}}[\mathrm{I}]$.

Similar results for anonymity and monotonicity.

Grandi and Endriss, Lifting Rationality Assumptions in Binary Aggregation, AAAI-2010.

Part IV:

Can We Avoid All Paradoxes?

How to Avoid all Paradoxes?

A generalised dictatorship copies the ballot of a (possibly different) individual (aka local dictatorships, positional dictatorships, rolling dictatorships):

Proposition

F is collectively rational with respect to all IC in $\mathcal{L}_{PS}$ if and only if F is a generalised dictatorship.

This class includes:

- Classical dictatorships $F(B_1, \ldots, B_n) = B_i$ for $i \in \mathcal{N}$
- Average Voter Rule: map $(B_1, \ldots, B_n)$ to the ballot B_i that minimises the sum of the Hamming distance to the others (the "average voter"). An interesting procedure!

The Average Voter Rule

Definition

The average voter rule (AVR) *chooses the individual ballot that minimises the sum of the Hamming distance* $H(B, B') = \sum_{j \in \mathcal{I}} |b_j - b'_j|$ *to all other individual ballots:*

$$\mathrm{AVR}(\mathbf{B}) = \operatorname*{argmin}_{\{B_i | i \in \mathcal{N}\}} \sum_{s \in \mathcal{N}} H(B_i, B_s),$$

The AVR shares interesting axiomatic properties:

Proposition

The AVR *satisfies a non-resolute version of anonymity, unanimity and monotonicity. It does not satisfy independence.*

Complexity of the AVR

$\textsc{WinDet}^\star(F)$
Instance: Integrity constraint IC, profile $\mathbf{B}$, issue j.
Question: Is there a $B \in F(\mathbf{B})$ such that $B_j = 1$?

Fact (very easy proof)

$\textsc{WinDet}^\star(\text{AVR})$ *is in P.*

Complexity of the AVR

WINDET$^\star(F)$	
Instance:	Integrity constraint IC, profile $\mathbf{B}$, issue j.
Question:	Is there a $B \in F(\mathbf{B})$ such that $B_j = 1$?

Fact (very easy proof)

WINDET*(AVR) *is in P.*

MANIPULABLE$^\star(F)$	
Instance:	IC, ballot B, partial profile $\mathbf{B}_{-i} \in \text{Mod}(\text{IC})^{|\mathcal{N}|-1}$.
Question:	Is there B' s.t. $H(B, F^\star(B', \mathbf{B}_{-i})) < H(B, F^\star(B, \mathbf{B}_{-i}))$?

Proposition

MANIPULABILITY(AVR) *is in P.*

More Results

More results can be obtained in this framework:

1. Computational complexity of safe aggregation: Π_2^p-complete in most cases we considered.
2. Classical impossibility results revisited: new flexible proof of Arrow's Theorem, agenda safety results in judgment aggregation.
3. More rules to avoid paradoxes: premise-based procedure and distance-based procedure.
4. Many more characterisation results on lifting rationality assumptions.

More information and detailed proofs:

Umberto Grandi, Binary Aggregation with Integrity Constraints, PhD Thesis, ILLC, 2012.

Grandi and Endriss. Binary Aggregation with Integrity Constraints, IJCAI-2011.

Part V:
Future Work

Ternary Aggregation?

Inviduals may be allowed to abstain:

- $\mathcal{D} = \{0, 1, A\}^{\mathcal{N}}$
- How to interpret integrity constraints?
- Characterisation results still hold?

More generally, integrity constraints may be expressed in an arbitrary logical language $\mathcal{L}$:

- $\mathcal{D} = \mathrm{Mod}(\mathcal{L})^{\mathcal{N}}$
- Integrity constraints are formulas $\varphi \in \mathcal{L}$
- How to perform the aggregation?

What are logical operators to merge profiles of models? (examples: direct products, ultraproducts for first-order logic)

Graph Aggregation

In preference aggregation individuals submit rankigs. Why not generalise this to directed graphs?

- Argumentation frameworks use attack graphs between conflicting arguments. In a multi-agent setting these graphs need to be aggregated.
- Social and economic networks represented as directed graphs can be merged.
- Preferences can be transitive, acyclic, partial...but always directed graphs (cf. Pini Et Al., Aggregating Partially Ordered Preferences, JLC, 2008)

Results from binary aggregation apply to this setting as well, and we studied the problem of collective rationality in this setting.

Endriss and Grandi, Graph Aggregation, COMSOC-2012.

- What about expressing integrity constraints in modal logic?
- Are there aggregation procedures that are peculiar to this setting?

Sequential Aggregation

When domains are combinatorial, preferences must be expressed compactly or cannot be fully elicited. This creates problems. A solution is to vote sequentially on one variable at the time: need for a "meaningful" order!

Airiau Et Al., Aggregating Dependency Graphs into Voting Agendas in Multi-Issue Elections, poster at IJCAI-2011.

Questions:

- Given integrity constraints/agenda/hard constraints defining a problem, can we devise a sequential order which avoids paradoxical situations?
- When individuals express preferences using CP-nets or preference representation languages, how to devise a sequential order of voting which minimises conflicts with the individual preferences?
- Can we devise a preference representation language which facilitates the creation of a sequential order of vote given individual preferences?

Summing Up

- Binary aggregation is the study of the aggregation of individual expressions described by means of binary variables.
- Rationality assumptions can be represented as propositional formulas, and thus classified and analysed in syntactic terms.
- Aggregation is collectively rational when rationality assumption is lifted from individual to collective level.

What we discover:

- Majority rule: all paradoxes feature a disjunction of size at least 3.
- Collective rationality wrt. syntactically defined languages corresponds to classical axioms from Social Choice Theory.
- To avoid all paradoxes select the individuals which best represent the view of the group (e.g., minimise a distance).

Thanks for your attention!