

Restricted Manipulation in Iterative Voting

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Good Manipulation, Bad Manipulation

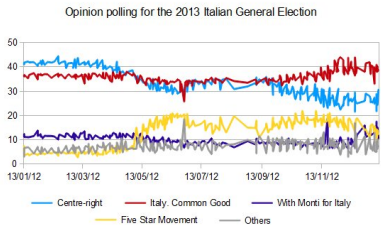
Manipulation in elections is usually considered a bad thing, to be **avoided** or at least to be made **computationally difficult** to achieve.



What if we can get a **better outcome** with **iterated manipulation** of simple rules, rather than complex-information-costly-almost-strategy-proof rules?

Practical Examples

In practice, **iterative manipulation** do occur:



Iterative response
to repeated polls

Table view | Calendar view | Australia/Melbourne

	Thu 25		Fri 24		Sat 26		Sun 27		Mon 28	
# participants	2:00 PM - 4:00 PM	9:00 AM - 11:00 AM	3:00 PM - 5:00 PM	9:30 AM - 11:30 AM	2:30 PM - 4:00 PM	9:30 AM - 11:30 AM	3:00 PM - 5:00 PM	9:00 AM - 11:00 AM	2:00 PM - 4:00 PM	
Participant 1	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Participant 2	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Participant 3	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Participant 4	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Participant 5	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Your name	Yes (Peak)	Yes (Peak)	Yes (Peak)	Yes (Peak)	Yes (Peak)	Yes (Peak)	Yes (Peak)	Yes (Peak)	Yes (Peak)	
Yes	5	6	5	6	5	4	4	7	6	
No	3	2	3	0	3	3	4	1	2	

Send

Approval voting with
iterative manipulation

Image source: Wikipedia, Doodle.com

Outline

1. The setting:

- Voting rules (in brief)
- Iterative voting
- Restricted manipulation: $M1$ and $M2$

2. Theoretical evaluation

- Convergence: **Yes!** (unknown for STV)
- Axiomatic properties: transfer to iterative rules

3. Experimental evaluation

- Condorcet efficiency: **Increase!**
- Average position of the winner: **Increase!**

Voting Rules

Things you all know:

- We start from a profile of **linear orders** over candidates $\{c_1, \dots, c_m\}$.
- **Positional Scoring Rules** give s_j points to candidates in position j in individual preferences, and elect the candidates with maximal score. We consider: Plurality, Borda, 2-approval, 3-approval, veto.
- **Copeland** elects the candidates which maximise the number of pairwise comparisons won minus the number of pairwise comparisons lost.
- **Maximin** elects the candidates with the highest minimal number of voters preferring her in pairwise comparisons.
- **Single Transferable Vote** deletes the candidate with the least first positions in individual preferences, transfer the votes to the succeeding candidate, and iterates until there is one candidate which has the majority of first positions.

Assumption: **linear** tie-breaking (for these slides $a > b > c > \dots$)

Iterative Voting

Manipulation occurs whenever a voter changes her ballot in her favour:

$$\begin{array}{ccc} a \succ b \succ c & & a \succ b \succ c \\ b \succ c \succ a & \longrightarrow & b \succ c \succ a \\ c \succ b \succ a & & \color{red}{b} \succ \color{red}{c} \succ a \\ \hline \text{Plurality: } a & & \text{Plurality: } b \end{array}$$

Definition

Given a set of manipulation moves M , a voting rule F (and a turn function) the iterated voting rule F^M associates with every profile \mathbf{b} the outcome of convergent iteration of manipulation moves in M (or \uparrow if it does not converge).

Unrestricted manipulation **does not always converge!**

R. Meir Et Al. Convergence to equilibria in plurality voting. AAI-2010.

O. Lev and J. S. Rosenschein. Convergence of iterative voting. AAMAS-2012.

Restricted Manipulation

Manipulation moves studied in the literature:

- **Best response** (no restriction): choose the ballot that changes the outcome of the election in the best way.
- **k-pragmatist**: put in first position your favourite candidate among the top k in the outcome of the voting rule.

A. Reijngoud and U. Endriss. Voter response to iterated poll information. AAMAS-2012.

Restricted Manipulation

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How to **evaluate** a manipulation move?

Convergence

Guaranteed
(small number of steps)

Computation

Not costly
(not NP-hard!)

Information

Low
(top candidate, scores..)

Restricted Manipulation: $M1$

Iteration starts at \mathbf{b}^0 (truthful) and continues to $\mathbf{b}^1, \dots, \mathbf{b}^k$ until convergence

$M1$

Move to the top the second-best candidate in b_i^0 (truthful), unless the current winner $w = F(\mathbf{b}^k)$ is already her best or second-best candidate in b_i^0 (truthful)

$a \succ b \succ c \succ d$		$a \succ b \succ c \succ d$		$a \succ b \succ c \succ d$
$c \succ b \succ a \succ d$		$b \succ c \succ a \succ d$		$b \succ c \succ a \succ d$
$d \succ b \succ c \succ a$	\rightarrow	$d \succ b \succ c \succ a$	\rightarrow	$b \succ d \succ c \succ a$
<hr/>		<hr/>		<hr/>
Plurality: a		Plurality: a		Plurality: b

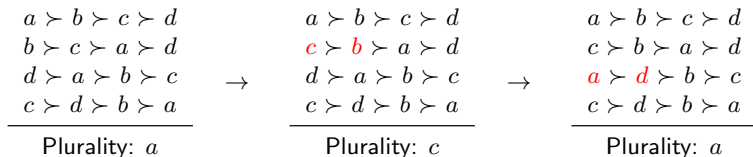
Minimal computation cost, minimal information required.

A side note: b is the **Condorcet winner**.

Restricted Manipulation: $M2$

$M2$

move to the top the best candidate in b_i^0 (truthful) which is above $w = F(\mathbf{b}^k)$ in b_i^k (reported), among those that can become the new winner of the election



Low computation cost, low information required (score, majority graph).

Convergence

Theorem

F^{M1} converges for every voting rule.

Proof idea: $M1$ can be applied only once by each individual.

Theorem

F^{M2} converges for *PSR*, *Copeland* and *Maximin*.

Proof idea: the score of the winner increases at every step, or remains the same and the candidate moves up in the tie-breaking order.

Axiomatic Properties

Axiomatic properties are preserved at every step of the iteration:

Theorem

M1 and M2 preserve unanimity.

If we start from a unanimous profile, the winner is always the top preferred candidate at every step of the iteration.

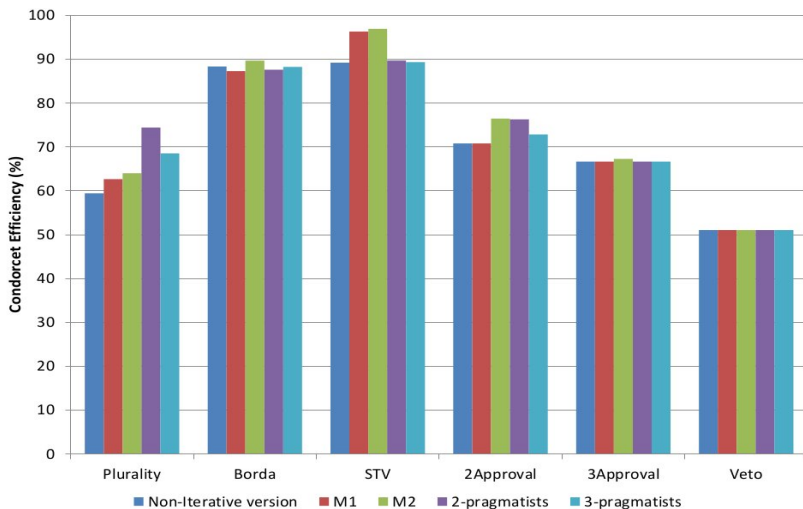
Theorem

M1 and M2 preserve Condorcet consistency.

Same for anonymity and neutrality. Pareto-condition does not transfer.

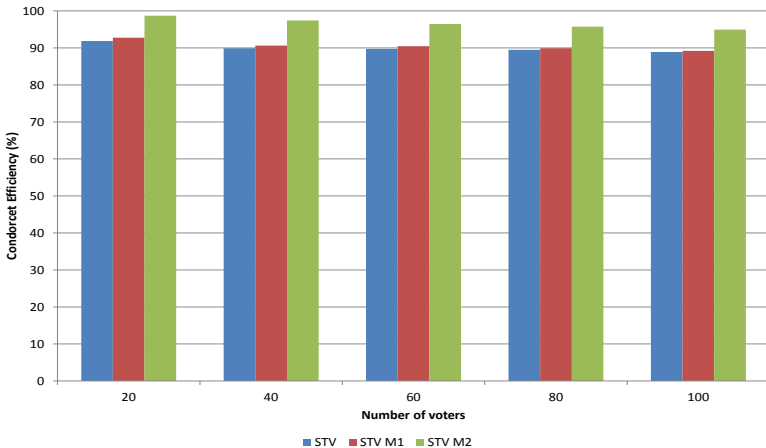
Condorcet Efficiency I

For Plurality better 2P and 3P, for all others $M2$ is better.
Positive performance of $M1$, even if little changes.



Condorcet Efficiency II

Higher efficiency for $n = 20$, stabilizes at around $n = 60$.
Consistently more than 95% for STV!



Motivational Intermezzo (M2)

One further motivation for iterated manipulation is that the Condorcet winner may be extracted without having to ask for the full profile.

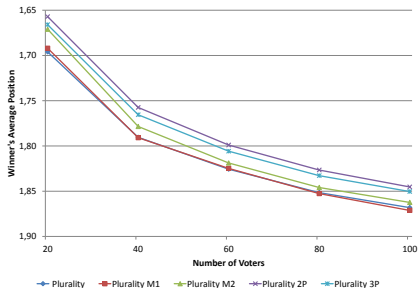
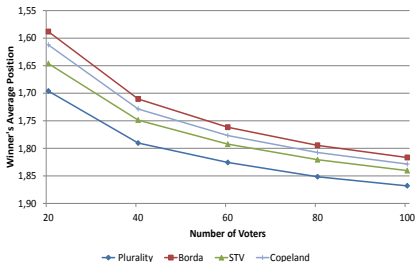
But: is it **more costly** to iterate or to ask for the full profile?

	# profiles with iteration	average # steps	maximal # steps
Plurality	2902	11.8	27
STV	1173	1.7	7
Borda	1961	8.1	31
2-Approval	2395	9.1	17

Profiles are 50×5 , maximal number of iterations is 27: good for Plurality!
Iteration takes place between 10% and 30% of the cases:
Not very costly, given the increase in Condorcet efficiency!

Average Position of the Winner

How much preferred is the winner in average?



For all voting rules (except for Borda) the position of the winner **increases** with iteration (and moves **towards the position of the Condorcet winner**).

Conclusions and Future Work

We introduced two new restricted manipulation moves which are **easy to compute** and need **small amount of information**, and we evaluated:

- Convergence of restricted iterative voting
- Condorcet efficiency
- Average position of the winner
- Number of iteration steps

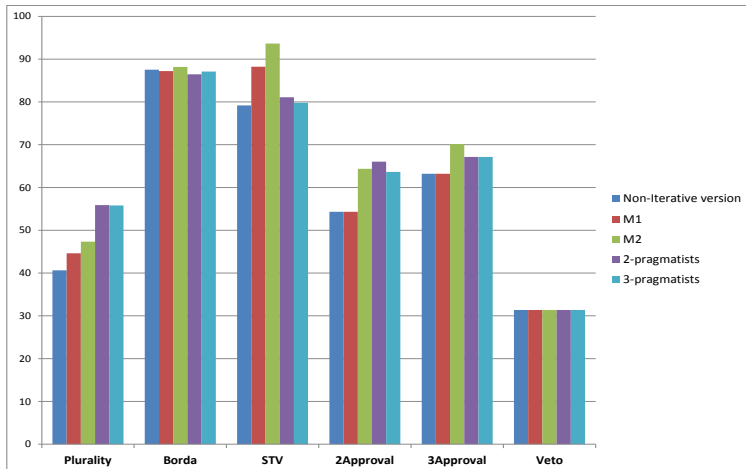
Restricted manipulation in iterative voting **increases** the Condorcet efficiency and the average position of the winner in a **limited number of steps**.

Lots of future questions:

- What is the best restricted manipulation move?
- Other parameters to judge iterated rules?
- More realistic distribution of preferences (no IC assumption).

Thank you for your attention!

Condorcet Efficiency



Condorcet Efficiency II

Higher efficiency for $n = 20$, stabilizes at around $n = 50$.

