

Restricted Manipulation in Iterative Voting: Convergence and Condorcet Efficiency

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Good Manipulation, Bad Manipulation

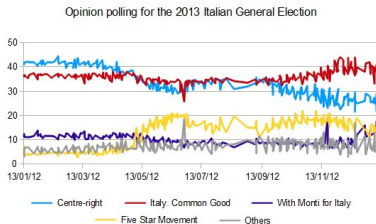
Manipulation in elections is usually considered a bad thing, to be **avoided** or at least to be made **computationally difficult** to achieve.



What if we can get a **better outcome** with **iterated manipulation** of simple rules, rather than complex-information-costly-almost-strategy-proof rules?

Practical Examples

In practice, **iterative manipulation** do occur:



Iterative response
to repeated polls

Table view | Calendar view | Australia/Melbourne

	Thu 25		Fri 24		Sat 26		Sun 27		Mon 28	
# participants	2:00 PM - 4:00 PM	9:00 AM - 11:00 AM	3:00 PM - 5:00 PM	9:30 AM - 11:30 AM	2:30 PM - 4:00 PM	9:30 AM - 11:30 AM	3:00 PM - 5:00 PM	9:00 AM - 11:00 AM	2:00 PM - 4:00 PM	
Participant 1	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Participant 2	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Participant 3	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Participant 4	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Participant 5	✓	✓	✓	✓	✓	✓	✓	✓	✓	
Your name	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	
	(Thru: 0)	(Thru: 0)	(Thru: 0)	(Thru: 0)	(Thru: 0)	(Thru: 0)	(Thru: 0)	(Thru: 0)	(Thru: 0)	
Yes	5	6	5	6	5	4	4	7	6	
No	3	2	3	0	3	3	4	1	2	

Send

Approval voting with
iterative manipulation

Image source: Wikipedia, Doodle.com

Outline

1. The setting

- Strategic manipulation and voting rules
- Voting games / iterative voting
- Restricted manipulation: $M1$ and $M2$

2. Theoretical and experimental evaluation

- Convergence: **Yes!** (unknown for STV)
- Axiomatic properties: transfer to iterative rules
- Condorcet efficiency and average position of the winner: **Increase!**

Voting Rules

What is a voting rule:

- We start from a set of individuals $1, \dots, n$ expressing preferences over candidates $\{c_1, \dots, c_m\}$: a **profile** $(\prec_1, \dots, \prec_n)$ of linear orders
- The voting rule uses this information to compute a (set of) **winner(s)**.

Examples of voting rules:

- **Positional Scoring Rules** give s_j points to candidates in position j in individual preferences, and elect the candidates with maximal score. We consider: Plurality, Borda, 2-approval, 3-approval, veto.
- **Copeland, Maximin** ...
- **Single Transferable Vote** deletes the candidate with the least first positions in individual preferences, transfer the votes to the succeeding candidate, and iterates until there is one candidate which has the majority of first positions.

Assumption: **linear** tie-breaking (for these slides $a > b > c > \dots$)

Strategic Manipulation

Manipulation occurs whenever a voter changes her ballot in her favour:

$$\begin{array}{ccc} a \succ b \succ c & & a \succ b \succ c \\ b \succ c \succ a & \longrightarrow & b \succ c \succ a \\ c \succ b \succ a & & \color{red}{b} \succ \color{red}{c} \succ a \\ \hline \text{Plurality: } a & & \text{Plurality: } b \end{array}$$

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Is there any chance to avoid manipulation?

Theorem [Gibbard-Satterthwaite]

Given a voting rule F , one of the following facts must be true: (i) there is a candidate that never wins (ii) F is a dictatorship, (iii) F can be manipulated.

Needless to say, all voting rules presented are manipulable...

A. Gibbard. Manipulation of voting schemes: A general result. *Econometrica*, 1973.

M. A. Satterthwaite, Strategy-proofness and Arrows conditions... *JET*, 1975.

Voting Games / Iterative Voting

Strategic manipulation in elections defines a **voting game**:

- Strategies are linear orders: individuals can change their preferences to obtain a better outcome
- The outcome is the result of the voting rule
- Utilities are defined by the truthful preferences of individuals

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Definition

Given a set of manipulation moves M , a voting rule F (and a turn function) the iterated voting rule F^M associates with every profile \mathbf{b} the outcome of convergent iteration of manipulation moves in M (or \uparrow if it does not converge).

Unrestricted manipulation **does not always converge!** But if it does, it converges to a **Nash equilibrium** of the voting game associated to F .

R. Meir Et Al. Convergence to equilibria in plurality voting. AAAI-2010.

O. Lev and J. S. Rosenschein. Convergence of iterative voting. AAMAS-2012.

Restricted Manipulation

Manipulation moves studied in the literature:

- **Best response** (no restriction): choose the ballot that changes the outcome of the election in the best way.
- **k-pragmatist**: put in first position your favourite candidate among the top k in the outcome of the voting rule.

A. Reijngoud and U. Endriss. Voter response to iterated poll information. AAMAS-2012.

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How to **evaluate** a manipulation move?

Convergence

Guaranteed
(small number of steps)

Computation

Not costly
(not NP-hard!)

Information

Low
(top candidate, scores..)

Restricted Manipulation: $M1$

Iteration starts at \mathbf{b}^0 (truthful) and continues to $\mathbf{b}^1, \dots, \mathbf{b}^k$ until convergence

$M1$

Move to the top the second-best candidate in b_i^0 (truthful), unless the current winner $w = F(\mathbf{b}^k)$ is already her best or second-best candidate in b_i^0 (truthful)

$a \succ b \succ c \succ d$		$a \succ b \succ c \succ d$		$a \succ b \succ c \succ d$
$c \succ b \succ a \succ d$		$b \succ c \succ a \succ d$		$b \succ c \succ a \succ d$
$d \succ b \succ c \succ a$	\rightarrow	$d \succ b \succ c \succ a$	\rightarrow	$b \succ d \succ c \succ a$
<hr/>		<hr/>		<hr/>
Plurality: a		Plurality: a		Plurality: b

Minimal computation cost, minimal information required.

A side note: b is the **Condorcet winner**.

Restricted Manipulation: $M2$

$M2$

move to the top the best candidate in b_i^0 (truthful) which is above $w = F(\mathbf{b}^k)$ in b_i^k (reported), among those that can become the new winner of the election

$\mathbf{a} \succ b \succ c \succ d$		$a \succ b \succ \mathbf{c} \succ d$		$a \succ b \succ c \succ d$
$b \succ c \succ \mathbf{a} \succ d$		$\mathbf{c} \succ b \succ a \succ d$		$c \succ b \succ a \succ d$
$d \succ \mathbf{a} \succ b \succ c$	\rightarrow	$d \succ a \succ b \succ \mathbf{c}$	\rightarrow	$\mathbf{a} \succ d \succ b \succ c$
$c \succ d \succ b \succ \mathbf{a}$		$\mathbf{c} \succ d \succ b \succ a$		$c \succ d \succ b \succ a$
<hr/>		<hr/>		<hr/>
Plurality: a		Plurality: c		Plurality: a

Low computation cost, low information required (score, majority graph).

Convergence and Axiomatic Properties

Theorem

Restricted iterated voting with $M1$ converges for every voting rule.

Proof idea: $M1$ can be applied only once by each individual.

Theorem

Restricted iterated voting with $M2$ converges for PSR, Copeland and Maximin.

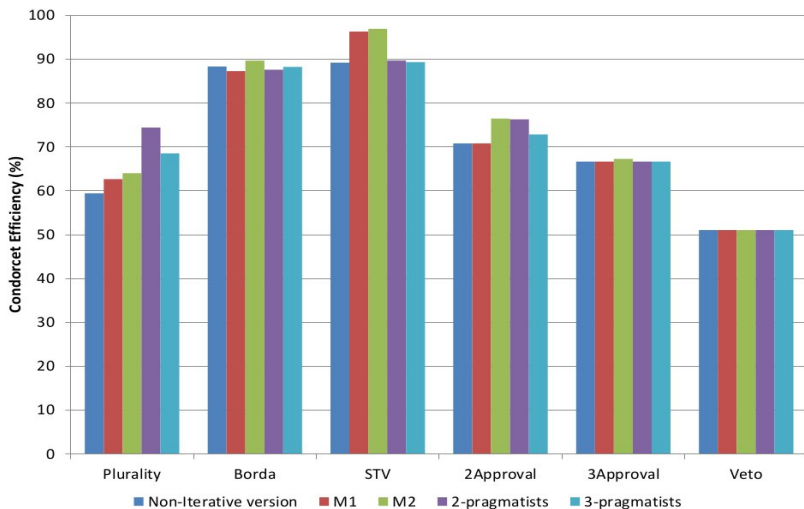
Proof idea: the score of the winner increases at every step, or remains the same and the candidate moves up in the tie-breaking order.

Axiomatic properties are **preserved** at every step of the iteration:

Example: if F is unanimous (elects a candidate if it is on top of every individual preference) then F^{M1} and F^{M2} are unanimous.

Condorcet Efficiency

For Plurality better 2P and 3P, for all others $M2$ is better.
Positive performance of $M1$, even if little changes.



Motivational Intermezzo (M2)

One further motivation for iterated manipulation is that the Condorcet winner may be extracted without having to ask for the full profile.

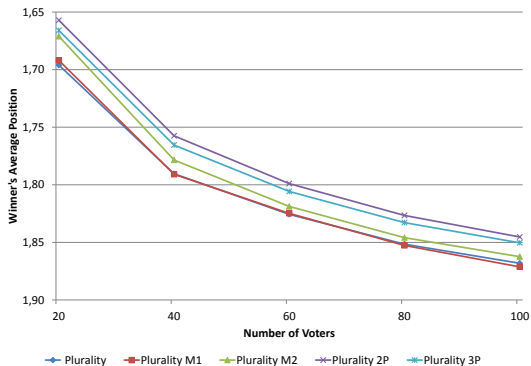
But: is it **more costly** to iterate or to ask for the full profile?

	# profiles with iteration	average # steps	maximal # steps
Plurality	2902	11.8	27
STV	1173	1.7	7
Borda	1961	8.1	31
2-Approval	2395	9.1	17

Profiles are 50×5 , maximal number of iterations is 27: good for Plurality!
Iteration takes place between 10% and 30% of the cases:
Not very costly, given the increase in Condorcet efficiency!

Average Position of the Winner

How much preferred is the winner in average?



For all voting rules (except for Borda) the position of the winner **increases**

Conclusions and Future Work

We introduced two new restricted manipulation moves which are **easy to compute** and need **small amount of information**, and we evaluated:

- Convergence of restricted iterative voting
- Condorcet efficiency
- Average position of the winner
- Number of iteration steps

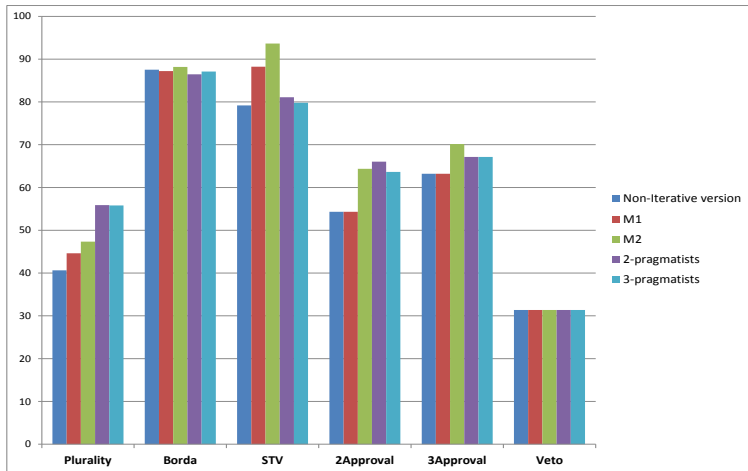
Restricted manipulation in iterative voting **increases** the Condorcet efficiency and the average position of the winner in a **limited number of steps**.

Lots of future questions:

- What is the best restricted manipulation move?
- Other parameters to judge iterated rules?
- More realistic distribution of preferences (no IC assumption).

Thank you for your attention!

Condorcet Efficiency



Condorcet Efficiency II

Higher efficiency for $n = 20$, stabilizes at around $n = 50$.

