

Pre-Vote Negotiations and Binary Voting with Constraints

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Logic and Rational Choice

Logic has met Economics (most notably game theory) in recent years:

Logic as a formal model of **rational behaviour**

- Logic in the foundations of economic theories of rational behaviour
- Choice models on logical expressions (judgment aggregation)
(this is the setting of this talk!)
- Related but not to this talk: game semantics in logic

Now add a computational touch to it:

Logic as a **compact language** to express properties
in a formal language that can (sometimes) be easily implemented

Everything Starts From Paradoxical Situations

Suppose three agents in a **multi-agent system** need to decide whether to perform a collective decision A . The decision is performed if two parameters T_1 and T_2 exceed a given threshold. Consider the following situation:

	T_1	T_2	A
Agent 1	Yes	Yes	Yes
Agent 2	No	Yes	No
Agent 3	Yes	No	No
Majority	Yes	Yes	No

Should the agents perform action A or not?

- A majority of agents think the first parameter exceeds the threshold.
- A majority of agents think the second parameter exceeds the threshold.
- **But:** a majority of agents think action A should not be performed!!

Binary Aggregation

Ingredients:

- A finite set \mathcal{N} of individuals
- A finite set $\mathcal{I} = \{1, \dots, m\}$ of **issues**
- A boolean **combinatorial domain**: $\mathcal{D} = \{0, 1\}^{\mathcal{I}}$

Definition

An aggregation procedure is a function $F : \mathcal{D}^{\mathcal{N}} \rightarrow \mathcal{D}$ mapping each profile of ballots $\mathbf{B} = (B_1, \dots, B_n)$ to an element of the domain \mathcal{D} .

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Example: Three agents with sensors

- $\mathcal{N} = \{1, 2, 3\}$
- $\mathcal{I} = \{T_1, T_2, A\}$
- Individuals submit ballots in $\mathcal{D} = \{0, 1\}^3$

$B_1 = (0, 1, 0)$ the first agent think the action should not be performed.

Integrity Constraints

A **propositional language** \mathcal{L} to express integrity constraints on $\mathcal{D} = \{0, 1\}^m$

- One propositional symbol for every issue: $PS = \{p_1, \dots, p_m\}$
- \mathcal{L}_{PS} closing under connectives $\wedge, \vee, \neg, \rightarrow$ the set of atoms PS

Given an integrity constraint $IC \in \mathcal{L}_{PS}$, a **rational** ballot is $B \in \text{Mod}(IC)$

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Example: Three agents with sensors

Perform action A if both parameters exceed the thresholds.

Propositional constraint: $IC = (p_{T_1} \wedge p_{T_2}) \rightarrow p_A$

Individual 1 submits $B_1 = (1, 1, 1)$: B_1 satisfies IC ✓

Individual 2 submits $B_2 = (0, 1, 0)$: $B_2 \models IC$ ✓

Individual 3 submits $B_3 = (1, 0, 0)$: $B_3 \models IC$ ✓

Majority aggregation outputs $(1, 1, 0)$: IC **not** satisfied.

Paradoxical situations cannot be avoided

The notorious **Condorcet** paradox in binary aggregation...

\triangle	\succ_1	\circ	\succ_1	\square		
\square	\succ_2	\triangle	\succ_2	\circ		
\circ	\succ_3	\square	\succ_3	\triangle		
<hr/>						
\triangle	\succ	\circ	\succ	\square	\succ	\triangle



	$\triangle \circ$	$\circ \square$	$\triangle \square$
Agent 1	1	1	1
Agent 2	0	1	0
Agent 3	1	0	0
<i>Maj</i>	1	1	0

Where are decisions actually taken?

Ok, paradoxes cannot be avoided...

...but we do take decisions!

How? Offline ("nei corridoi", in italiano)



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Vision

Pre-vote dynamics

create consensus and avoid paradoxes!

– *deliberation, negotiation, bribery, power and authority, opinion polls* –

Dryzek and List, Social Choice Theory and Deliberative Democracy: A reconciliation. *BJPS*, 2003.

List, Group Communication and the Transformation of Judgment. *JPP*, 2011.

Structure of the results

We model collective decision making as a **game**:

- players have preferences/goals over collective outcomes
- strategies are voting ballots
- framework of **boolean games**

Question

Can a paradoxical situation be a Nash equilibrium?

A Nash equilibrium (NE) is a profile of strategies/ballots (B_1, \dots, B_n) in which no player has incentives to deviate by changing her strategy.

Harrenstein, van der Hoek, Meyer, Witteveen. Boolean games. *TARK-2001*.

Aggregation Games

Definition [Aggregation games]

An aggregation game is a tuple $\mathcal{A} = \langle \mathcal{N}, \mathcal{I}, \text{IC}, \text{maj}, \{\gamma_i\}_{i \in \mathcal{N}} \rangle$ such that:

- $\langle \mathcal{N}, \mathcal{I}, \text{IC} \rangle$ is a binary aggregation structure;
- maj is the majority rule;
- the individual goals γ_i are IC-consistent propositional formulas in \mathcal{L}_{PS} ;

Focus on **consistent** game: $\bigwedge_i \gamma_i \wedge \text{IC}$ is consistent:

Good news: there always exists an IC-consistent equilibrium, i.e., a profile B in which no individual has incentive to deviate and $\text{maj}(B) \models \text{IC}$

Bad news: there are games with **inefficient NE** (no individual goal satisfied)

Very bad news: there are games with **inefficient and IC-inconsistent NE**

Aggregation Games with Payoff

Let us give individuals more expressive preferences:

- goals γ_i represent uncompromising positions
- any two states which both satisfy or both falsify the goal γ_i can be compared by looking at the payoff function π_i

Definition [A^π games]

An aggregation game with payoff is a tuple $\langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ where \mathcal{A} is an aggregation game and $\pi_i : \text{Mod}(\text{IC})^{\mathcal{N}} \rightarrow \mathbb{R}$ is a payoff function.

Bad news: inefficient and IC inconsistent equilibria still exists...

Good news: for every consistent aggregation game \mathcal{A} we can build payoff functions such that $\langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ has **no inefficient NE!**

Endogenous Aggregation Games

We model pre-vote negotiations in two phases:

- A **pre-vote phase**: starting from a uniform A^π -game players make simultaneous transfers of payoff to their fellow players
- A **vote phase**: players play the original A^π -game, updated with transfers

Definition [A^T -games]

An endogenous aggregation game is defined as a tuple $\langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}}, \{T_i\}_{i \in \mathcal{N}} \rangle$ where $\langle \mathcal{A}, \{\pi_i\}_{i \in \mathcal{N}} \rangle$ is a uniform A^π game, and each T_i is the set of all transfer functions $\tau_i : \text{Mod}(\text{IC})^{\mathcal{N}} \times \mathcal{N} \rightarrow \mathbb{R}_+$

Pre-vote Negotiation avoids Inefficient Equilibria

A NE B is **surviving** if there are transfer functions τ_i that **sustain it**: (τ, B) must form an equilibria of the two-phase game.

Proposition

Every goal-efficient NE of an \mathcal{A}^T -games is a surviving NE.

Proposition

Every surviving NE of a consistent \mathcal{A}^T -games is goal-efficient.

Good news: individuals can redistribute payoff to reach goal-efficient NE!

Are we avoiding paradoxes?

It is possible to avoid paradoxes:

Corollary

Every consistent \mathcal{A}^T -game has an IC-consistent NE that is surviving.

If at least one individual has coherence as a goal it is also guaranteed:

Corollary

Every surviving NE of an \mathcal{A}^T -game such that $\bigwedge_i \gamma_i \models \text{IC}$ is IC-consistent.

Conclusions

1. Collective decisions on logically interconnected issues **can be paradoxical**
Notorious examples: Condorcet paradox, discursive dilemma...
2. When voters are strategic (read self-interested) the collective decision at equilibrium may be **undesirable or unrealistic**
3. **Pre-vote negotiation** rules out undesirable equilibria!
 - The model: two phase-games, first negotiate then vote
 - Main result: surviving equilibria iff they satisfy the goals of each voter
 - Corollary: there always exists a consistent equilibria which is surviving

If you want to know more:

U. Grandi, D. Grossi, P. Turrini. Pre-vote Negotiations and Binary Voting with Constraints.
11th Conference on Logic and the Foundations of Game and Decision Theory, 2014.

ESSLLI 2014 course on "Logical Frameworks for Multiagent Aggregation"
Tübingen, 11–15 August — **Student grants available!**