Binary Aggregation with Integrity Constraints

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A General Framework for Aggregation

Preference Aggregation
Condorcet Paradox
Arrow's Theorem

Judgment Aggregation
Doctrinal paradox
Agenda properties

Voting in Combinatorial Domains
Paradox of multiple elections
...

Binary Aggregation with Integrity Constraints
Paradox of collective rationality
Characterisation results
(from Grandi and Endriss AAAI-2010)

Binary aggregation with integrity constraints constitutes a general framework for the study of paradoxes and (im)possibility results in aggregation.

Outline

1. Basic definitions: binary aggregation and integrity constraints
2. Definition of paradox: Condorcet, Discursive Dilemma...
3. Collective rationality and a new proof method for (im)possibility results
4. Conclusion and generalised dictatorships

Binary Aggregation

Ingredients:

- A finite set N of individuals
- A finite set $\mathcal{I} = \{1, \ldots, m\}$ of issues
- A boolean *combinatorial domain*: $\mathcal{D} = D_1 \times \cdots \times D_m$ with $|D_i| = 2$

Definition

An aggregation procedure is a function $F : \mathcal{D}^N \to \mathcal{D}$ mapping each profile of ballots $\underline{B} = (\underline{B}_1, \ldots, \underline{B}_n)$ to an element of the domain $\mathcal{D}$.

Dokow and Holzman (2005), Grandi and Endriss (AAAI-2010)

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Example: Town Council

- $N = \{a_1, a_2, a_3\}$
- $\mathcal{I} = \{\text{Shopping mall}, \text{Train station}, \text{Road}\}$
- Individuals submit ballots in $\mathcal{D} = \{0, 1\}^3$

$B_1 = (0, 1, 0)$ individual 1 only approves the construction of a train station.

Integrity Constraints

- One propositional symbol for every issue: $PS = \{p_1, \ldots, p_m\}$
- $\mathcal{L}_{PS}$ closing under connectives $\wedge$, $\vee$,$\neg$, $\rightarrow$ the set of atoms PS

An integrity constraint is a formula IC $\in \mathcal{L}_{PS}$
The set of rational ballots is $\mathrm{Mod}(\mathsf{IC}) \subseteq \mathcal{D}$

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Example: Town Council (with congested roads)

If there is both a train station and a shopping mall then a new road is necessary
Propositional constraint: IC $= (p_S \wedge p_T) \rightarrow p_R$

Individual 1 submits $B_1 = (1, 0, 0)$: B_1 satisfies IC ✓
Individual 2 submits $B_2 = (1, 1, 1)$: $B_2 \models$ IC ✓
Individual 3 submits $B_3 = (0, 1, 0)$: $B_3 \models$ IC ✓

Majority aggregation outputs $(1, 1, 0)$: IC not satisfied (as are all drivers)

Paradoxes of Aggregation: Definition

Every individual satisfies the same rationality assumption IC...
...what about the collective outcome?

Definition

A paradox is a triple $(F, \underline{B}, IC)$, where:

- F *is an aggregation procedure*
- $\underline{B} = (B_1, \ldots, B_n)$ *a profile*
- $IC \in \mathcal{L}_{PS}$ *an integrity constraint*

such that $B_i \models IC$ *for all* $i \in \mathcal{N}$ *but* $F(\underline{B}) \not\models IC$.

Paradoxes of Aggregation: Instances

Preference Aggregation:

Completeness and antisymmetry: $p_{ab} \leftrightarrow \neg p_{ba}$ for $a \neq b \in \mathcal{X}$

Transitivity: $(p_{ab} \wedge p_{bc}) \rightarrow p_{ac}$ for $a, b, c \in \mathcal{X}$ pairwise distinct

Condorcet Paradox: (majority rule, profile in table, $\mathrm{IC}_<$)

	ab	bc	ac
Agent 1	1	1	1
Agent 2	0	1	0
Agent 3	1	0	0
Majority	1	1	0

Judgment Aggregation:
Integrity constraints IC_Φ encoding consistency of judgment set

Discursive Dilemma: (majority rule, profile in table, IC_Φ)

Others: Ostrogorski paradox, multiple-elections, town council...

Collective Rationality and Characterisation Results

Definition

F is collectively rational (CR) for IC $\in \mathcal{L}_{PS}$ if for all profiles $\underline{B}$ such that $\underline{B}_i \models$ IC for all $i \in N$ then $F(\underline{B}) \models$ IC.

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Collective rationality with respect to propositional sublanguages can be characterised using classical axiomatic properties:

Characterisation Result

An aggregation procedure F is collectively rational with respect to any cube (i.e. conjunction of literals) if and only if it is unanimous.

Other results:

U. Grandi and U. Endriss. Lifting Rationality Assumptions in Binary Aggregation, AAAI-2010.

Impossibility Results: New Proof Method

Search for clashes between integrity constraints and axiomatic properties:

Proposition

Any anonymous, independent and monotonic social welfare function for more than 3 alternatives and 2 individuals is imposed.

Proof sketch:

- Axiom preserving correspondence from social welfare functions to binary aggregation procedures collectively rational for $\mathrm{IC}_<$
- Characterisation result: A, I, M procedures are CR wrt IC iff it satisfies a certain property or procedure is imposed
- the integrity constraints $\mathrm{IC}_<$ does not satisfy this property

How to avoid all paradoxes?

A generalised dictatorship copies the ballot of a (possibly different) individual (aka local dictatorships, positional dictatorships, rolling dictatorships):

Proposition

F is collectively rational with respect to all IC in $\mathcal{L}_{PS}$ if and only if F is a generalised dictatorship.

This class includes:

- Classical dictatorships $F(B_1, \ldots, B_n) = B_i$ for $i \in \mathcal{N}$
- Distance-based generalised dictatorship: map $(B_1, \ldots, B_n)$ to the ballot B_i that minimises the sum of the Hamming distance to the others (the "average voter"). An interesting procedure!

Conclusion

Binary aggregation with integrity constraints:

- Language to express rationality assumptions
- Concept of collective rationality with respect to an IC
- General framework for paradoxes (e.g., Condorcet and doctrinal)
- New proof method for impossibility results: clash between axioms and IC
- Interesting procedures: distance-based generalised dictatorship

Future work:

- Explore connection with belief merging and argumentation theory
- More complex domains: full combinatorial domains, logical languages for integrity constraints other than propositional
- Binary aggregation and voting: approval voting, aggregation of preferential dependency graphs (cf. Airiau et al. *Aggregating Dependency Graphs into voting Agendas in Multi-Issue Elections*, IJCAI-11)