Quanto Implied Correlation in a Multi-Lévy Framework

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Extended abstract

The objective of this paper is to introduce a flexible and tractable multivariate FX model based on Lévy processes capturing the fluctuations in both the foreign exchange rate and equity log-returns. Further, we show how the proposed model can be used in conjunction with market quotes of actively traded Quanto futures to back out the parameters describing the risk neutral joint dynamics of FX rate and equity log-returns, and therefore information about correlation and tail dependence. As Quanto products offer significant exposure to the correlation between exchange rates and asset prices, they allow access to a market implied measure of this correlation.

The interest in these models is motivated by the fact that FX risk and its joint evolution with the many other risk factors present in portfolios of financial institutions and insurance companies can adversely affect these portfolios, and consequently the corresponding capital requirements, as quantified by risk measures such as Value at Risk (VaR) [see 4, for example]. Further, as the empirical evidence provided for example by [3] shows that the risk neutral conditional distribution of currency returns is leptokurtic and its skewness changes over time, the standard framework based on the Brownian motion, i.e. the Gaussian distribution, is not realistic enough for a correct portrayal of the distribution tails behaviour. Moreover, this latter feature
is of primary importance for the computation of relevant risk measures, as these aim at quantifying the economic impact of rare events. A simple but effective way of replacing the Gaussian distribution is the introduction of jumps by adopting Lévy processes, as many analytical formulas established for models based on the Brownian motion can be easily extended to this more general class of processes. We note at this stage two interconnected issues related to the discussion above. In first place, portfolio risk measures can usually be computed only numerically; hence the ease of implementation of any given model is a desirable feature to have. Secondly, risk measures need to be computed in a common (domestic) currency, which puts particular emphasis on the availability of reliable information regarding the dependence structure in place between the relevant risk factors. To this aim, we observe that Quanto products, such as Quanto futures and Quanto options represent convenient instruments written on this interdependence. These are, in fact, financial products with a payoff paid in a different currency from the one in which the asset is traded, allowing investors to participate in the asset profit without facing any exposure to foreign exchange rate risk. Quanto futures in particular are actively traded in the market and therefore quotes are supported by sufficient liquidity to generate reliable estimates.

Due to its balance between flexibility (meant as dimension of the parameter space) and mathematical tractability (meant as availability of the relevant characteristic functions), in this paper we apply the factor model of [2] to build a multivariate FX framework, which also includes an additional asset to cater for the underlying asset of Quanto products such as Quanto futures and Quanto options. The construction is therefore based on the assumption of a common source of systematic risk in both stock and foreign exchange returns, which is consistent with the results of [1]. This also allows us to develop an integrated calibration procedure which provides access to information on the dependence structure between the relevant components. We point out that although our framework is based on the factor construction of [2], in which convolution conditions required to recover a known distribution for the margin processes are derived and applied, our model does not need these restrictive conditions, as they are not necessary to retain its mathematical tractability and a limited number of parameters.

In particular, the present paper offers the following contributions. Firstly, we show that the proposed multivariate FX model satisfies symmetries with respect to inversion and triangulation; further, it gives access to analytical formulae for the correlation coefficient and the indices of tail dependence. Secondly, our model leads to analytical results (up to a Fourier inversion) for the price of both vanilla and Quanto options, therefore the model can be
easily calibrated to market quotes. Thirdly, the application of the proposed model to the pricing of Quanto futures reveals that the quanto adjustment is not only determined by the covariance between asset log-returns (as in the standard Black-Scholes model), but also by higher order cumulants of the jump part of the systematic risk. As these are an explicit function of the parameters of the systematic process, market consistent information on the (in general not observable) common component can be extracted directly from the market, bypassing the need of either imposing unrealistic convolution conditions, or identifying a suitable proxy for this part of the risk. As the same quanto adjustment also enters the pricing formulas of Quanto options, the proposed model allows us to compare the information on the existing correlation recovered from Quanto futures with the one backed out of Quanto option prices and the one extracted from the relevant time series, i.e. the historical correlation commonly used by practitioners in the market. We apply these ideas to the USD-denominated Quanto type futures on the Nikkei 225 index to extract the correlation between the USDJPY FX rate and the Nikkei 225 index log-returns. The results show that whilst correlation values implied by Quanto futures and Quanto options are relatively similar, they differ significantly from historical correlation, regardless of the time horizon considered for sampling; this discrepancy has an important impact on risk measures for portfolios containing such products, such as VaR and indices of tail dependence. Finally, the empirical analysis shows that the quanto adjustment is particularly affected by the covariance between FX and stock log-returns, and the skewness of the distribution of the common source of systematic risk.

Keywords
FX risk, implied correlation, multivariate Lévy processes, Quanto products, Variance Gamma process.

References

