AMBIGUOUS GAMES WITHOUT A STATE SPACE AND FULL RATIONALITY

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Abstract

There is a strand of research in decision theory in which the decision-maker, facing ambiguity, is not able to understand what the relevant states are so that the information available can be expressed entirely in the space of probabilities (lotteries) over consequences. This approach without the state space and the classical approach, characterized by multiple priors on a state space, can be reconciled by a form of probabilistic sophistication where an ambiguous act is evaluated by its induced set of distributions over consequences; however, game theory provides further evidence that ambiguity cannot always be reconducted to the classical approach with a state space and multiple priors as there is a specific source of ambiguity in games when players have ambiguous beliefs about opponents’ strategy choices.

This paper studies strategic form games under ambiguity in which, for every player, ambiguity is directly represented by a belief correspondence which maps the set of strategy profiles to the set of all subsets of probability distributions over the outcomes of the game. Belief correspondences give the set of probability distributions over the possible outcomes of the game that the corresponding players perceive to be feasible and consistent with the actual strategy profile. On the one hand, belief correspondences might represent objective (exogenous) ambiguity over consequences; on the other hand, it turns out that many existing models of ambiguous game have an equivalent formulation in terms of belief correspondences. For example, incomplete information games with multiple priors on the state space and the partially specified equilibrium concept can both be regarded as particular cases of our notion of equilibrium under ambiguous belief correspondences.

In this framework, a pessimistic (resp. optimistic) decision maker would not prefer a set of probability distributions over outcomes to another one if the former is a subset of the latter (resp. the latter is a subset of the former). We build upon these behavioral assumptions of minimal character to obtain equilibrium existence theorems. The main result of this paper is that equilibria exist with pessimistic (resp. optimistic) players provided that the ambiguous belief is a convex (resp. concave) set-valued map. The role of the assumptions of the existence theorem is studied in specific models in order to illustrate result’s applicability.