Approximated pricing of swaptions in general interest rate models

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Extended abstract

The accurate pricing of swaption contracts is fundamental in interest rate markets, but modelling swaption payoff may be relevant also beyond the standard setting. For example Basel III accords introduced the Credit Value Adjustment (C.V.A.) charge for over the counter contracts. It is interesting that for the most simple and popular kind of interest rate derivative, i.e. interest rate swap, the (unilateral) C.V.A. can be estimated as a portfolio of forward start European swaptions. We propose new bounds on the prices of European-style swaptions for a wide class of interest rate models. These bounds are computable whenever the joint characteristic function of the state variables is known in closed form or can be obtained numerically via some efficient procedure. In particular our lower bound involves the computation of one dimensional Fourier transform independently from the swap length. This allows a reduction of the computational cost, mainly when we have to price swaptions written on long-maturity swaps. We also show that methods put forward by Singleton and Uman\cprime sev [19] and Kim [13] are particular cases of our general framework. Indeed, we prove that their
approximations are also lower bounds to the swaption price. This has gone completely unnoticed up to now. In addition, we control the error of our method by providing a new upper bound on swaption price applicable to all linear-quadratic models. In literature upper bounds are available only for Gaussian affine models. Finally the lower bound can be used as a control variable to reduce the confidence interval of the Monte Carlo technique. We test our bounds on different affine models, also allowing for jumps, and on a 2-factors quadratic Gaussian model. The bound are found to be accurate and computationally efficient.

**Keywords**
Swaption; Pricing; Lower Bound; Upper bound; Fourier Transform.

**References**


