Extended abstract

In a population consisting of heterogeneous types, whose income factors are indicated by nonnegative vectors, policies aggregating different factors can be represented by coalitions in a cooperative game, whose characteristic function is a multi-factor inequality index. When it is not possible to form all coalitions, the feasible ones can be indicated by a graph.

In this framework, differently from the typical constructions in Cooperative Game Theory where most games have a payoff function (see many examples in [2]), a cooperative game on the factor set $\mathcal{F}$ can be seen as a cost game where the cost of factor-driven inequality is associated to each coalition. When dealing with a graph structure $G = (\mathcal{F}, E)$ whose feasible set is $G_x$, we will denote with $I_G(S)$ the characteristic cost function measuring inequality of the feasible coalition $S$ of the game on $G$.

We are going to take into account the two most famous standard allocations, i.e. the Shapley value and the Banzhaf value (see [2]) and then reformulate them to apply them to connected graphs in order to determine the income factors’ marginal contributions to overall inequality.

We analyze graphs with the following structure:
by using a specific multi-factor inequality index.

If $Y \in \mathbb{R}_+^I$ is an income distribution whose factors are $F_1, \ldots, F_M$ and $\epsilon \in (0, 1)$ is an aversion parameter, the multi-factor Atkinson index of inequality, $I_A(Y_{F_1, \ldots, F_m})$ is:

$$I_A(Y_{F_1, \ldots, F_m}) = 1 - \frac{1}{\mu_j} \sum_{j=1}^{M} \tilde{y}_j = 1 - \frac{N^{-\frac{1}{1-\epsilon}}}{M} \sum_{j=1}^{M} \left( \frac{\left[ \sum_{i=1}^{N} y_{ij}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}}{\sum_{i=1}^{N} y_{ij}} \right). \quad (1)$$

An example is finally provided based on a modified multi-factor Atkinson index.

**Keywords**
Inequality Index; Shapley Value; Graph.

**References**


