HIERARCHICAL PRINCIPAL–AGENT PROBLEMS IN CONTINUOUS–TIME

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1. The one-period model

2. The continuous-time model
   A similar framework but in continuous-time
   Resolution of the two Stackelberg equilibria

3. Numerical Results

4. Conclusion and extensions
THE ONE–PERIOD MODEL

A hierarchical Principal–Agent model in one–period with moral hazard.

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The Agents are the $n+1$ risk–averse workers of the firm (with CARA utility). Each Agent $i \in \{0, \ldots, N\}$ (he) produces the random outcome $X^i$ by carrying out his own task:

$$X^i = \alpha^i + \sigma^i W^i,$$

where $W^i \sim \mathcal{N}(0, 1)$ are i.i.d.

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\]

where \( W_i \sim \mathcal{N}(0,1) \) are i.i.d.

The effort of the \( i \)–th Agent is the variable \( \alpha_i \), inducing him a cost \( c^i(\alpha^i) \geq 0 \).
Direct contracting: the Principal offers a contract at time $t = 0$ for each Agent to incentivise them to act in her best interest at time $t = 1$, i.e. to improve the benefit of the firm.
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- Interlinked Principal–Agent problems – Sequence of Stackelberg equilibria.
SEQUENCE OF STACKELBERG EQUILIBRIA

Figure: Sung’s Model
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▶ It is common in one–period models to restrict the study to linear contracts:

\[
\xi^i = \xi_0^i - \sup_{a \in \mathbb{R}} \left\{ aZ^i - c^i(a) \right\} + Z^iX^i + \frac{1}{2}R^i(Z^i)^2\text{Var}(X^i),
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where \( Z^i \) is a parameter chosen by the Manager.
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where \( Z^i \) is a parameter chosen by the Manager.

▶ **Optimal effort**: \( \hat{\alpha}^i(Z^i) \).
The Manager controls the mean and the variance of his state variable $\zeta$.

$$\zeta = \alpha^0 + \sigma^0 W^0 - \sum_{i=1}^{n} \left( \xi_i^0 - \hat{\alpha}^i(Z^i) + c^i(\hat{\alpha}^i(Z^i)) + \frac{1}{2} R^i(Z^i \sigma^i)^2 \right)$$

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$$\zeta = \alpha^0 + \sigma^0 W^0 - \sum_{i=1}^{n} \left( \xi_0^i - \tilde{\alpha}^i(Z^i) + c^i(\tilde{\alpha}^i(Z^i)) + \frac{1}{2} R^i(Z^i \sigma^i)^2 \right)$$

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But, in continuous–time with volatility control, linear contracts are not optimal, see Cvitanić, Possamaï, and Touzi 2018...
THE CONTINUOUS–TIME MODEL
The $i$–th Agent

- controls the drift of a process $X^i$ with dynamic $dX^i_t = \alpha^i_t dt + \sigma^i_t dW^i_t$;
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- controls the drift of a process $X^i$ with dynamic $dX^i_t = \alpha^i_t dt + \sigma^i_t dW^i_t$;
- receives a terminal payment $\xi^i$ which is a function of $(X^i)_{t \in [0,1]}$.
A SIMILAR FRAMEWORK BUT IN CONTINUOUS–TIME

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The Manager

- controls the drift of a process $X^0$ with dynamic $dX^0_t = \alpha^0_t dt + \sigma^0_t dW^0_t$;
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- receives a terminal payment $\xi^0$.

The Principal only observes in continuous–time the process $\zeta$

$$\zeta_t = \sum_{i=0}^n X^i_t - \sum_{i=1}^n \xi^i_t,$$

for $t \in [0,1]$, and indexes the contract $\xi^0$ for the Manager on it.
The $i$–th Agent:

\[ V^i_0(\xi^i) := \sup_{\alpha^i} \mathbb{E}^{\nu^i} \left[ - \exp \left( - R^i \left( \xi^i - \int_0^1 c^i(\alpha^i_t)dt \right) \right) \right]. \]

We will assume for simplicity that $c^i(a) = a^2/2k^i$ (quadratic costs).
VALUE FUNCTIONS

The $i$–th Agent:

$$V_i^0(\xi^i) := \sup_{\alpha^i} \mathbb{E}^{p_i}\left[ - \exp \left( - R^i \left( \xi^i - \int_0^1 c^i(\alpha^i_t) dt \right) \right) \right].$$

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The Manager:

$$V_0^0(\xi^0) := \sup_{\alpha^0, (\xi^i)_{i=1,\ldots, n}} \mathbb{E}^{p_0}\left[ - \exp \left( - R^0 \left( \xi^0 - \int_0^1 c^0(\alpha^0_t) dt \right) \right) \right]$$
**Value Functions**

The $i$–th Agent:

$$V_i^0(\xi^i) := \sup_{\alpha^i} \mathbb{E}^{ip_i} \left[ - \exp \left( - R^i \left( \xi^i - \int_0^1 c^i(\alpha^i_t) \, dt \right) \right) \right].$$

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The Principal:

$$V_0 = \sup_{\xi^0} \mathbb{E}^{ip^*} [\xi_1 - \xi^0_1].$$
Assumption: the compensation for the i-th Agent can only be indexed on his own outcome process $X^i$. 
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The **optimal** form of contracts for the $i$–th Agent is (see Sannikov 2008):

$$
\xi^i = \xi_0^i - \int_0^1 \mathcal{H}^i(Z^i_s)ds + \int_0^1 Z^i_s dX^i_s + \frac{1}{2} R^i \int_0^1 (Z^i_s)^2 d\langle X^i \rangle_s,
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$$

where

(i) $Z^i$ is a payment rate chosen by the Manager;
(ii) $\mathcal{H}^i(z) = \sup_{a \in \mathbb{R}} \{az - c^i(a)\}$ is the $i$–th Agent’s Hamiltonian.
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(ii) $H^i(z) = \sup_{a \in \mathbb{R}} \{az - c^i(a)\}$ is the i–th Agent’s Hamiltonian.

The optimal effort of the i–th Agent is $\alpha^i_t = k^i Z^i_t$, and we can compute the dynamics of $X^i$ and $\xi^i$ with this optimal effort.
The Manager controls $\alpha^0$ and $Z^i$, for $i \in \{1, \ldots, n\}$. 

Assumption: the Principal only observes $Z^i$ in continuous-time, where:

$$
\begin{aligned}
&d_t = 0_t dt + 0_t dW_t + \sum_{i=1}^{n} (k_i Z^i_t - (k_i + R^i_t))^2 dt + \sum_{i=1}^{n} (1 - Z^i_t) dW^i_t;
\end{aligned}
$$

and thus its quadratic variation (see Bichteler 1981).

▶ The Manager controls the volatility of his state variable.

▶ By Cvitanić, Possamaï, and Touzi, 2018, the optimal form of contracts is:

$$
\begin{aligned}
&0_t = \int_0^1 H_0(Z_s; s) ds + \int_0^1 Z_s ds + \frac{1}{2} \int_0^1 (Z_s + R_0 Z^2_s) d\left\langle Z \right\rangle_s;
\end{aligned}
$$

(2)
The Manager controls $\alpha^0$ and $Z^i$, for $i \in \{1, \ldots, n\}$.

**Assumption:** The Principal only observes $\zeta$ in continuous-time, where:

$$d\zeta_t = \alpha_t^0 dt + \sigma_t^0 dW_t^0 + \sum_{i=1}^n \left( k^i Z_t^i - \frac{1}{2} (Z_t^i)^2 \left( k^i + R^i (\sigma^i)^2 \right) \right) dt$$

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&\quad + \sigma^i \sum_{i=1}^{n} \left( 1 - Z^i_t \right) \, dW^i_t,
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- By Cvitanić, Possamaï, and Touzi 2018, the **optimal** form of contracts is:

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\xi^0 = \xi^0_0 - \int_0^1 \mathcal{H}^0(\zeta_s, \Gamma_s) \, ds + \int_0^1 Z_s \, d\zeta_s + \frac{1}{2} \int_0^1 \left( \Gamma_s + R^0 \zeta_s^2 \right) d\langle \zeta \rangle_s. \quad (2)
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(ii) the optimal control on the $i$–th Agent’s compensation is

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We can compute the dynamics of $\zeta$ and $\xi^0$ under optimal efforts.
The Principal’s problem is reduced to

\[ V_0 = \sup_{(Z, \Gamma) \in \mathbb{R}^2} \mathbb{E}^{\mathbb{P}^0} \left[ \xi_T - \xi_T^0 \right]. \]
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\[ V_0 = \sup_{(Z, \Gamma) \in \mathbb{R}^2} \mathbb{E}_P^0 \left[ \zeta_T - \xi_T^0 \right]. \]

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- The optimal \( \Gamma \) is different from Sung 2015 where he forced \( \Gamma = -R^0Z^2 \).

- We can write the optimal contracts designed by the Principal to the Manager, and by the Manager to each Agent.
NUMERICAL RESULTS
Figure: Effort of the Manager depending on the number of Agents.
... TO DECREASE THE AGENTS’ EFFORT

Figure: Effort of an Agent depending on the number of Agents.
GAIN IN UTILITY FOR THE PRINCIPAL

Figure: Value function of the Principal depending on the number of Agents.
CONCLUSION AND EXTENSIONS
We improve the results of Sung 2015 by moving to continuous-time, since it allows to add a quadratic variation term in the contract for the Manager.

This model can be extended to:

(i) a more general hierarchy;
(ii) other forms of reporting;
(iii) adding an "ability" parameter of the Manager.

Extend to a more general model (work in progress) with:

(i) general output dynamics;
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