HIERARCHICAL PRINCIPAL–AGENT PROBLEMS IN CONTINUOUS–TIME

Emma Hubert[1],
with the relevant ideas and useful advice of Dylan Possamaï (Columbia University).

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OUTLINE

1. The one–period model
2. The continuous–time model
   - A similar framework but in continuous–time
   - Resolution of the two Stackelberg equilibria
3. Numerical Results
4. Conclusion and extensions

THE ONE–PERIOD MODEL

Sung 2015 – Pay for performance under hierarchical contracting.

- A hierarchical Principal–Agent model in one–period with moral hazard.

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The Agents are the $n+1$ risk–averse workers of the firm (with CARA utility). Each Agent $i \in \{0, \dots, N\}$ (he) produces the random outcome X^i by carrying out his own task:

$$X^i = \alpha^i + \sigma^i W^i,$$

where $W^i \sim \mathcal{N}(0,1)$ are i.i.d.

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The effort of the i–th Agent is the variable α^i, inducing him a cost $c^i(\alpha^i) \geq 0$.

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▶ Interlinked Principal–Agent problems – Sequence of Stackelberg equilibria.

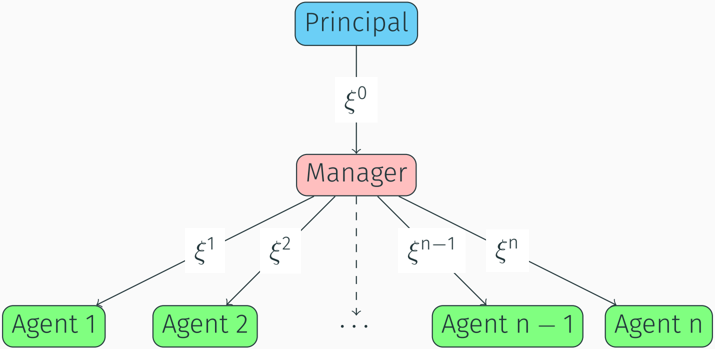

Figure: Sung's Model

DOUBLE MORAL HAZARD

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$$\zeta := \sum_{i=0}^{n} X^i - \sum_{i=1}^{n} \xi^i.$$

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- The contract ξ^0 for the Manager is indexed on ζ.

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$$\xi^i = \xi_0^i - \sup_{a \in \mathbb{R}} \left\{ a Z^i - c^i(a) \right\} + Z^i X^i + \frac{1}{2} R^i \left(Z^i\right)^2 \mathbb{V}\mathrm{ar}(X^i),$$

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where Z^i is a parameter chosen by the Manager.

▶ Optimal effort: $\widehat{\alpha}^i(Z^i)$.

- The Manager controls the mean and the variance of his state variable ζ.

$$\zeta = \alpha^0 + \sigma^0 W^0 - \sum_{i=1}^{n}\left(\xi_0^i - \widehat{\alpha}^i(Z^i) + c^i(\widehat{\alpha}^i(Z^i)) + \frac{1}{2}R^i(Z^i\sigma^i)^2\right) + \sum_{i=1}^{n}(1 - Z^i)\sigma^i W^i.$$

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- The variance of ζ is not observed by the Principal, and thus the contract cannot be indexed on it.
- Sung 2015 restrict again the study to linear contracts, without the variance term.
- But, in continuous–time with volatility control, linear contracts are not optimal, see Cvitanić, Possamaï, and Touzi 2018…

THE CONTINUOUS–TIME MODEL

The $i-$th Agent

- controls the drift of a process X^i with dynamic $dX_t^i = \alpha_t^i dt + \sigma^i dW_t^i$;

A SIMILAR FRAMEWORK BUT IN CONTINUOUS–TIME

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- controls the drift of a process X^i with dynamic $dX^i_t = \alpha^i_t dt + \sigma^i dW^i_t$;
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The Principal only observes in continuous–time the process ζ

$$\zeta_t = \sum_{i=0}^{n} X^i_t - \sum_{i=1}^{n} \xi^i_t,$$

for $t \in [0, 1]$, and indexes the contract ξ^0 for the Manager on it.

The $i-$th Agent:

$$V_0^i(\xi^i) := \sup_{\alpha^i} \mathbb{E}^{\mathbb{P}^i}\left[-\exp\left(-R^i\left(\xi^i - \int_0^1 c^i(\alpha_t^i)\mathrm{d}t\right)\right)\right].$$

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The Principal:

$$V_0 = \sup_{\xi^0} \mathbb{E}^{\mathbb{P}^\star}\left[\zeta_1 - \xi_1^0\right].$$

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where

(i) Z^i is a payment rate chosen by the Manager;

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▶ The optimal effort of the i–th Agent is $\widehat{\alpha}_t^i = k^i Z_t^i$, and we can compute the dynamics of X^i and ξ^i with this optimal effort.

RESOLUTION OF THE PRINCIPAL–MANAGER PROBLEM (1)

The Manager controls α^0 and Z^i, for $i \in \{1, \ldots, n\}$.

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- By Cvitanić, Possamaï, and Touzi 2018, the **optimal** form of contracts is:

$$
\xi^0 = \xi_0^0 - \int_0^1 \mathcal{H}^0(Z_s, \Gamma_s) ds + \int_0^1 Z_s d\zeta_s + \frac{1}{2} \int_0^1 (\Gamma_s + R^0 Z_s^2) d\langle \zeta \rangle_s. \quad (2)
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- We can the compute the dynamics of ζ and ξ^0 under optimal efforts.

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RESOLUTION OF THE PRINCIPAL PROBLEM

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▶ We can write the optimal contracts designed by the Principal to the Manager, and by the Manager to each Agent.

NUMERICAL RESULTS

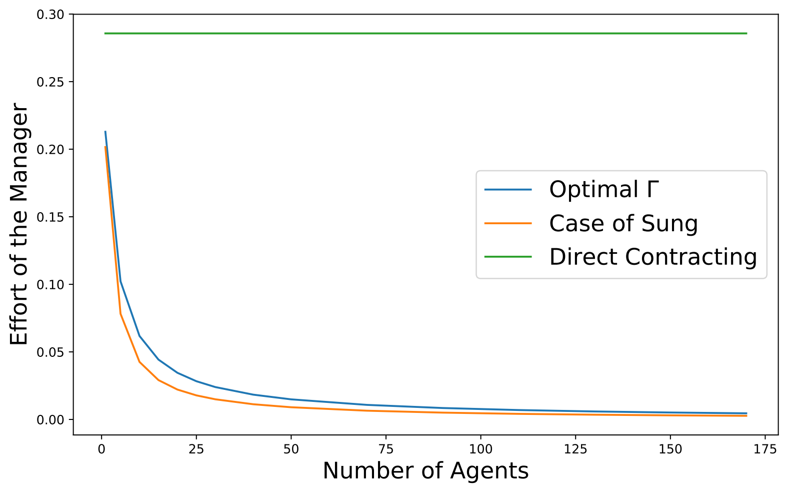

Figure: Effort of the Manager depending on the number of Agents.

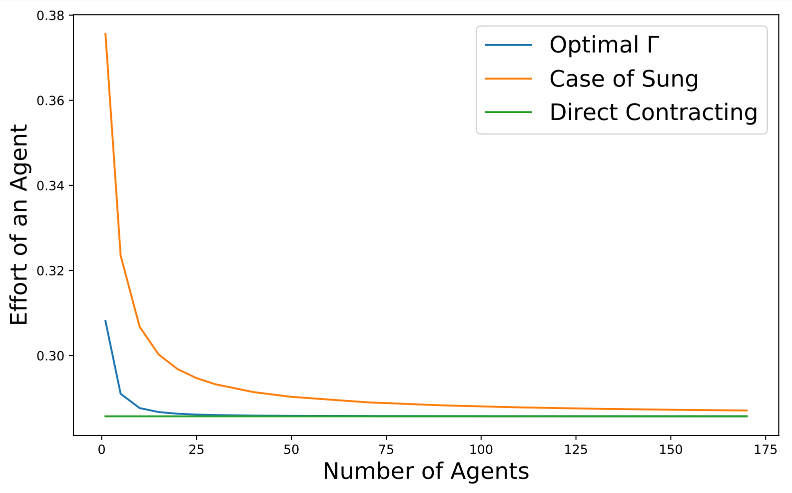

Figure: Effort of an Agent depending on the number of Agents.

GAIN IN UTILITY FOR THE PRINCIPAL

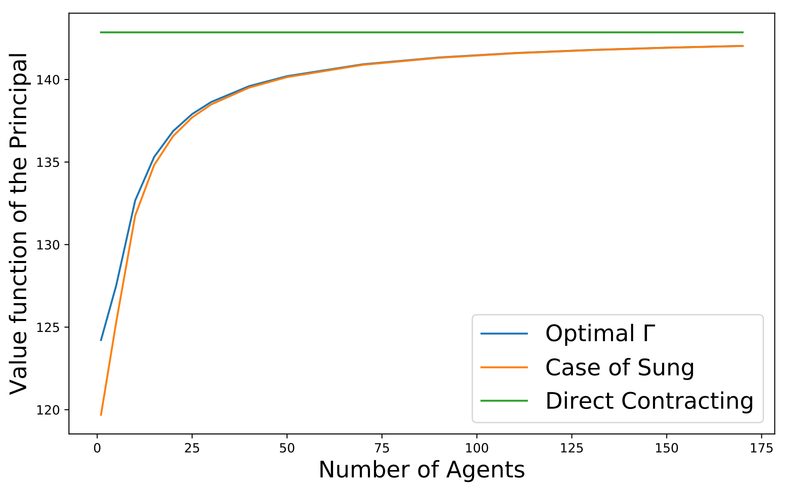

Figure: Value function of the Principal depending on the number of Agents.

CONCLUSION AND EXTENSIONS

- We improve the results of Sung 2015 by moving to continuous–time, since it allows to add a quadratic variation term in the contract for the Manager.

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(i) a more general hierarchy;
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(iii) adding an "ability" parameter of the Manager.

- Extend to a more general model (work in progress) with:

(i) general output dynamics;
(ii) general utility functions;
(iii) general cost functions;
(iv) general form of reporting ζ.

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